Date:
Registration number:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
M.Sc. MATHEMATICS - II SEMESTER
END SEMESTER EXAMINATION: APRIL 2022
(Examination conducted in July 2022)

MT 8218 – COMPLEX ANALYSIS

Time- 2 ½ hrs Max Marks-70

This question paper contains **TWO** printed sides.

Answer any 7 questions: **7*10=70**

1. State and prove Cauchy's theorem in a circular disc. **[10m]**
2. Let $\sum_{n=0}^{\infty} a_n z^n$ be a given power series. Let $A = \lim_{n\to\infty} |a_n|^{1/n}$ and let $R = 1/A$ then prove that
   i. If $0 < A < \infty$ then $0 < R < \infty$ holds and the power series converges for all z with $|z| \leq R$ and the series diverges for $|z| > R$.
   ii. If $A = 0$ then $R = \infty$ and the series $\sum_{n=0}^{\infty} a_n z^n$ converges for all $z \in \mathbb{C}$.
   iii. If $A = \infty$ then $R = 0$ and the series $\sum_{n=0}^{\infty} a_n z^n$ diverges for all $z \neq 0$. **[10m]**
3. Find the Laurent's series expansion for $f(x) = \frac{z}{(z-1)(z-3)}$ when
   i. $|z| < 1$ ii. $|z-1| < 2$ **[5m+5m]**
4. A. If f and g are entire functions and $|f(z)| \leq |g(z)|, \forall\, z \in \mathbb{C}$, prove that $f(z) = c.g(z)$ for some constant c.
   B. Find the residue of $f(z) = \frac{z^2}{(z-1)^2(z+2)}$. **[6m+4m]**
5. A. Using Cauchy's Residue Theorem evaluate $\int_c \frac{cosz}{z\,(z-1)^2}\, dz$, where $c : |z| = 3$.
   B. Discuss the singularity of the function $f(z) = \frac{1-e^z}{1+e^z}$ at $z = \infty$. **[5m+5m]**
6. A. Evaluate $\int_0^{2\pi} \frac{d\theta}{1-2pcos\theta+p^2}, 0 < p < 1$.
   B. Define Zero of a function. **[8m+2m]**
7. A. If n is a positive integer then show that $\int_0^{2\pi} \sin(n\theta - sin\theta)\, e^{cos\theta}\, d\theta = 0$
   B. Evaluate $\int_{-\infty}^{\infty} \frac{cosx}{x^2+a^2}\, dx, a > 0$ **[5m+5m]**
8. State and prove Hadamard's Three Circle Theorem. **[10m]**
9. A. State and prove Phragmen- Lindelöf Theorem.
   B. If $|z| \leq 1$ and $p \geq 0$ then show that $|1 - Ep(z)| \leq |z|^{p+1}$. **[6m+4m]**
10. State and prove Poisson's Jensens Formula. **[10m]**