ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
B.Sc. MATHEMATICS - II SEMESTER
SEMESTER EXAMINATION: APRIL 2022
(Examination conducted in July 2022)
MT 221: MATHEMATICS II

Time: 2 Hours   **Max Marks:** 60

This question paper contains **TWO** printed pages and **FIVE** parts.

I. ANSWER ANY <u>SIX</u> OF THE FOLLOWING.   (6×2=12)

1. Find the identity element of the group $(\mathbb{Z}, *)$ with $a * b = a + b + 1$.

2. Without computing the order explicitly, show that 2 and 28 have same order in $(Z_{30}, +_{30})$.

3. Evaluate $\int\limits_{0}^{1} x^2(1-x)^{\frac{3}{2}} dx$

4. Find the area enclosed by the parabola $y^2 = 4ax$ and its latus rectum.

5. Find the angle between the radius vector and the tangent to the curve $r = a\ sin\theta$.

6. Find $\dfrac{ds}{dx}$ for the curve $x = a(t + sint)$ and $y = a(1 - cost)$.

7. Find the integrating factor of $(1 + x^2)\dfrac{dy}{dx} + y = e^{tan^{-1}x}$

8. Test for exactness and hence solve $(e^y + 1)\cos x dx + e^y \sin x dy = 0$

II. ANSWER ANY <u>TWO</u> OF THE FOLLOWING.   (2×6=12)

9. Show that $U(10)$ is a group under multiplication modulo 10 using cayley table.

10. State and prove two step subgroup test.

11. a) Define Order of an element of a group. Define order of a group and what is the order of the group $(\mathbf{R}, +)$?

  b) Write the order of each element of the group $(\mathbb{Z}_{10}, +_{10})$ (2+4)

III. ANSWER ANY <u>TWO</u> OF THE FOLLOWING.   (2×6=12)

12. a) Evaluate $\int\limits_{0}^{\pi} x \sin^4 x \cos^6 x dx$.

  b) Evaluate $\int\limits_{0}^{1} \dfrac{x^6}{\sqrt{1 - x^2}} dx$.   (4+2)

13. Obtain the entire length of the cardioid $r = a(1 + cos\theta)$.

14. Find the area bounded by the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$, $0 \leq \theta \leq 2\pi$ and its base.

IV. ANSWER ANY <u>TWO</u> OF THE FOLLOWING. $(2 \times 6 = 12)$

15. Find the angle of intersection for the following curve $r = \sin\theta + \cos\theta, r = 2\sin\theta$

16. Find the pedal equation of the curve $y^2 = 4a(x + a)$

17. Show that for the ellipse, $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, the radius of curvature is $\rho = \dfrac{a^2 b^2}{p^3}$

V. ANSWER ANY <u>TWO</u> OF THE FOLLOWING. $(2 \times 6 = 12)$

18. Solve $\dfrac{dy}{dx} + x \sin 2y = x^3 \cos^2 y$

19. Solve $\dfrac{dy}{dx} - 2y \, tanx = y^2 \, tan^2 x.$

20. Find the suitable integrating factor and solve the equation $xydx - (x^2 + 2y^2) \, dy = 0$