Registration Number:
Date & session:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27

M.Sc. (MATHEMATICS) - III SEMESTER

SEMESTER EXAMINATION: OCTOBER 2022

(Examination conducted in December 2022)

MT 9222 – Classical and Continuum Mechanics

Time: 2.5 Hours **Max Marks: 70**

Answer any SEVEN full questions from the following each carrying TEN marks:

1. Derive the expression for velocity and acceleration in spherical co-ordinate system.
2. (a) The position vector of two point masses 10kg and 5kg are (-3,2,4) and (3,6,5) respectively. Find the position of the center of mass.
   (b) State and prove principle of virtual work.
   (c) Derive the expression for generalized momentum for a system of particles. (2+5+3)
3. (a) Obtain Lagrangian form of D'Alembert's principle.
   (b) Define conservative force. Find the relation between work and kinetic energy. (4+6)
4. Derive the Lagrange's equation of motion for a holonomic dynamical system.
5. Define a conservative system and prove that the conditions for a conservative system are sufficient for the existence of an energy integral.
6. (a) Show that $(a_{ijk} + a_{jki} + a_{kij})x_i x_j x_k = 3a_{ijk}x_i x_j x_k$.
   (b) Show that $\delta_{ij}a_{kj} = a_{ki}$.
   (c) If $D = det(a_{ij})$ and $\varepsilon_{ijk}\varepsilon_{pqr}\, D = \begin{vmatrix} a_{ip} & a_{iq} & a_{ir} \\ a_{jp} & a_{jq} & a_{jr} \\ a_{kp} & a_{kq} & a_{kr} \end{vmatrix}$ then show that $\varepsilon_{ijk}\varepsilon_{pjk} = 2\delta_{ip}$ and $\varepsilon_{ijk}\varepsilon_{ijk} = 6$. (2+3+5)
7. (a) Find $(x_m\, x_n),_{ij}$.
   (b) If $\vec{v} = \emptyset\vec{u}$ then find $div(\vec{v})$.
   (c) State and prove Gauss Divergence theorem for a tensor. (3+2+5)

8. (a) Derive the relation between the surface element in the initial and final configuration.

   (b) Check if the given deformation is isochoric where the deformation is defined by equations $x_1^0 = \frac{1}{3}(2x_1 + x_2),\ x_2^0 = \frac{1}{3}(x_1 - x_2),\ x_3^0 = x_1 - x_3$ (5+5)

9. (a) Find the vertex line of the flow defined by $\vec{v} = (1 + at)\hat{e_1} + x_1\hat{e_2}$ where a is a constant.

   (b) If acceleration is a gradient of a potential then prove that the circulation around a closed curve as it moves around the fluid remains constant with respect to time. (5+5)

10. (a) Show that $\rho = \rho_0\, e^{-t^2}$, for a given velocity field $\vec{v} = (x_2\hat{e}_2 + x_3\hat{e}_3)t$

    (b) Derive the conservation of angular momentum. (4+6)
