Register Number:

Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
B.Sc. MATHEMATICS-V SEMESTER
SEMESTER EXAMINATION: OCTOBER-2022
(Examination to be conducted in December 2022)
MT-5118: MATHEMATICS V

Duration: 2.5 Hours **Max. Marks: 70**

The paper contains TWO pages and THREE parts

I. ANSWER ANY FIVE OF THE FOLLOWING. **(5x2=10)**

1. List all the units of the ring $(\mathbb{Z}_8, \oplus_8, \otimes_8)$.
2. List all the zero-divisors of the ring $(\mathbb{Z}_6, \oplus_6, \otimes_6)$.
3. Define prime ideal and maximal ideal.
4. Check if the mapping $f : (\mathbb{Z}, +, \cdot) \to (\mathbb{Z}_n, \oplus_n, \otimes_n)$ defined by $f(x) = x(mod\, n)$ is a ring homomorphism.
5. Find a_0 in the Fourier series expansion of the function $f(x) = \dfrac{x}{\pi^2}$ in the interval $(-\pi, \pi)$.
6. Find a_n in the Fourier series expansion of the function $f(x) = 1$ in the interval $(-\pi, \pi)$
7. Evaluate $\displaystyle\int_0^\infty x^6 e^{-x} dx$.
8. Evaluate $\displaystyle\int_0^{\pi/2} \sin^3\theta \cos^5\theta d\theta$.

II. ANSWER ANY SEVEN OF THE FOLLOWING. **(7x6=42)**

9. Define the centre of a ring and state the subring test. Using the subring test prove that centre of a ring R is a subring of R. .
10. Let R be a commutative ring with unity and b be a nilpotent element of R. Then prove that
    (a) $1 + b$ is a unit.
    (b) if a is a unit, then $a + b$ is a unit. [3+3]
11. Define an integral domain. Prove that cancellation laws hold good in commutative ring R with unity if and only if R is an integral domain.
12. Define characteristic of a ring. Prove that the characteristic of an integral domain D is either 0 or a prime.
13. Prove that $n\mathbb{Z}$ is prime ideal of $\mathbb{Z}$ if and only if n is a prime.
14. Let R be a commutative ring with unity and let A be an ideal of R. Prove that $\dfrac{R}{A}$ is a field if and only if A is maximal.

15. Define ring homomorphism and ring isomorphism. Let R be a commutative rig with characteristic 2. Show that $\phi : R \to R$ defined by $\phi(a) = a^2$ is a ring homomorphism.

16. Define automorphism. Show that $f : \mathbb{C} \to \mathbb{C}$ defined by $f(z) = \overline{z}$, is an automorphism. What is the $ker f$.

17. (a) Show that $\phi : \mathbb{R}[x] \to \mathbb{R}$ defined by $\phi(f(x)) = f(1), \forall f(x) \in \mathbb{R}[x]$ is a ring homomorphism. Find $ker\phi$.
    (b) Show that $\phi : \mathbb{Z} \to 3\mathbb{Z}$ defined by $\phi(a) = 3a$ is not a ring homomorphism. [4+2]

III. ANSWER ANY THREE OF THE FOLLOWING. **(3x6=18)**

18. Find the Fourier series expansion of the function f(x)=$1 - x^2$ in the interval $(-1, 1)$.

19. (a) Express $f(x) = x(\pi - x)$ as a half range Fourier sine series in the interval $(0, \pi)$.
    (b) Express $f(x) = x^3$ as a half range Fourier cosine series in the interval $(0, \pi)$. [2+4]

20. Show that $\beta(m, n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

21. Show that $\Gamma(n) = \dfrac{\Gamma(n+1)}{n}$ and hence evaluate $\Gamma\left(\dfrac{-7}{2}\right)$

22. (a) Evaluate $\beta\left(\dfrac{9}{2}, \dfrac{7}{2}\right)$
    (b) Evaluate $\displaystyle\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$ [2+4]