Register Number:
DATE:

**ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27**
**M.Sc. PHYSICS – III SEMESTER**
**SEMESTER EXAMINATION: OCTOBER 2022**
**(To be conducted in December 2022)**
**PH 9318/PH9320 : MODERN OPTICS**

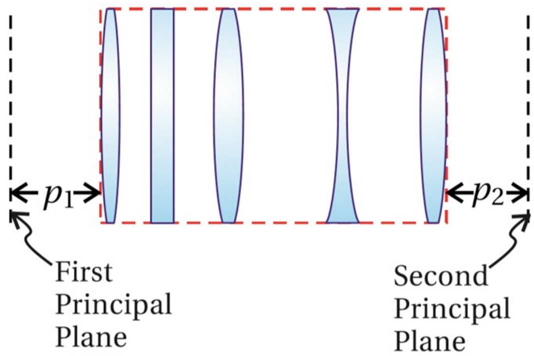
**Time: 2 1/2 hours** **Max Marks:70**

**This paper contains 2 parts and 3 printed pages.**

## Part-A

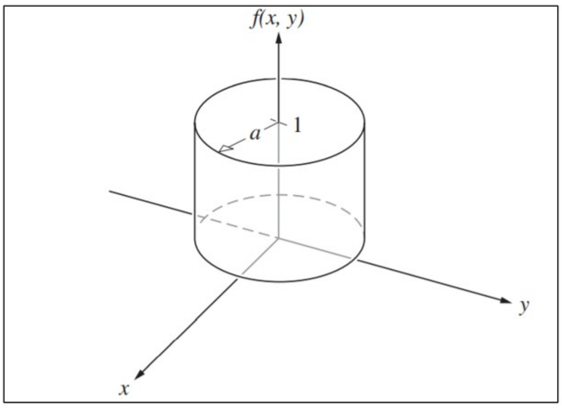
**Answer any FIVE questions. Each carries 10 marks.** **(5x10=50)**

1. a) Assuming the system matrix for the complex optical system shown in figure below to be $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ in general, append the matrices for $p_1$ and $p_2$ appropriately with it. Hence, deduce the expressions for $p_1$ and $p_2$. Show that for our choice of $p_1$ and $p_2$ , the assumption made to write the equation for calculating these values is correct.
   b) What is the need for finding the positions of these principal planes? (8+2)




2. The field distribution at a point on the screen for a single slit Fraunhofer diffraction pattern is given as $E = A\dfrac{\sin\beta}{\beta}\cos(\omega t - \beta)$ where $\beta = \dfrac{\pi b \sin\theta}{\lambda}$ , $b$ being the width of the slit and $\theta$ being the angle that diffracted ray makes with the normal to the grating. Using this result, derive an expression for the resultant intensity obtained from a diffraction grating made up of N such slits, separated by distance d between consecutive slits and interpret your result.

3. Find the Fourier transform of cylindrical function (as shown in figure) given as: $f(x,y) = \begin{cases} 1 & \sqrt{x^2+y^2} \le a \\ 0 & \sqrt{x^2+y^2} > a \end{cases}$



You can start with 2-dimensional Fourier transform equation: $F(k_x, k_y) = \iint\limits_{-\infty}^{+\infty} f(x, y) e^{i(k_x x + k_y y)} dxdy$ and use spherical polar coordinates to simplify it. Interpret the result explaining how is it relevant to Fraunhofer diffraction.

[Given: The Bessel function of (first kind) order zero is given as $J_o(u) = \frac{1}{2\pi}\int_0^{2\pi} e^{i\,u\cos\theta} d\theta$ and use this recurrence relation for the Bessel function $\frac{d}{du}[u^m J_m(u)] = u^m J_{m-1}(u)$ to relate $J_o(u)$ to $J_1(u)$ ].

4. a)Explain how spatial dependence of light wave is ensured by the experimental configuration of Michelson's Interferometer.

   b)If this interferometer is illuminated by red cadmium light source with mean wavelength of 643.85 nm and has finite linewidth then with relevant diagram explain how will you find the spectral width of the source(explain the method by first deriving the relevant expression)? What initial adjustment do you need to do? (2+8)

5. For a plane wave propagating inside an anisotropic medium, starting with the Maxwell's equations in material medium show that $\vec{D} = \frac{1}{\omega}(\vec{H} \times \vec{k})$ $\vec{H} = \frac{1}{\omega \mu_o}(\vec{k} \times \vec{E})$ . Now, solving these equations further to explain the wave propagation, arrive at the set of three homogenous equations given as:

$$\left(\frac{k_x}{n^2} - \kappa_y^2 - \kappa_z^2\right) E_x + \kappa_x \kappa_y E_y + \kappa_x \kappa_z E_z = 0$$

$$\kappa_y \kappa_x E_x + \left(\frac{k_y}{n^2} - \kappa_x^2 - \kappa_z^2\right) E_y + \kappa_y \kappa_z E_z = 0$$

$$\kappa_z \kappa_x E_x + \kappa_z \kappa_y E_y + \left(\frac{k_z}{n^2} - \kappa_x^2 - \kappa_y^2\right) E_z = 0$$

here $k_x, k_y, k_z$ give the principal refractive indices along the three principal axes $x, y, z$ respectively with propagation vector $\vec{k} = k\hat{\kappa}$ . $\kappa_x, \kappa_y, \kappa_z$ are components of the unit vector $\hat{\kappa}$ and $E_x, E_y, E_z$ are the components of the electric field in the principal axes system.

6. The inhomogeneous wave equation for the propagation of EM wave in a non-linear medium is given as $\nabla^2 E - \mu_o \epsilon \frac{\partial^2 E}{\partial t^2} = \mu_o \frac{\partial^2 P^{NL}}{\partial t^2}$ . The incident EM wave at frequency $\omega$ generates field at frequency $2\omega$ within this non-linear medium, with incident field given as $E = E_o \cos(kz - \omega t)$ and $P^{NL} = \epsilon_o \chi^{(2)} E^2$ . Starting with the fields at frequency $\omega$ and $2\omega$ and writing them in terms of sum of the field in complex form and its complex conjugate, arrive at the phase matching condition for second harmonic generation.

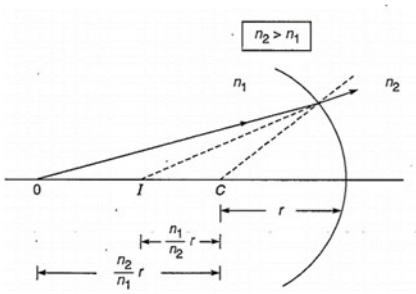
7. a) Using Huygen's principle, draw and explain how an unpolarized wave will propagate through a uniaxial crystal when the optic axis is parallel to the crystal surface on which light is incident. Hence, explain the principle of Babinet's compensator.

   b) Knowing that the condition for constructive interference in a reflection grating is $d(\sin i + \sin r) = n\lambda$ where $i$ is the angle of incidence of light and $r$ is the angle of reflection, explain the dispersive power and light gathering power of this grating. (6+4)

## Part-B

**Answer any FOUR questions. Each question carries 5 marks.** **(5X4=20)**

8. Consider a spherical refracting surface as shown in the figure. Using ABCD matrix analysis, show that for an object at a distance of $\left(1+\frac{n_2}{n_1}\right)r$ from the surface, the image formed is virtual and at a distance of $\left(1+\frac{n_1}{n_2}\right)r$ from the surface.




9. An achromatic doublet is used to make the objective lens of a telescope which has an effective focal length of 1.5 m. The materials used are crown glass with refractive index for C and F lines of hydrogen spectra as 1.5165 and 1.5249 respectively and flint glass with refractive index for C and F lines as 1.5742 and 1.5886 respectively. Find the focal length of these component lenses.

10. Michelson's stellar interferometer is used to measure the angular dimension of a star. If the fringes show successive minima for a displacement of 18 cm of one of the mirrors, calculate the angular dimension of the star assuming $\lambda$ to be 540 nm.

11. Find the Fourier transform of the function $A(1+\cos k_o x)$ . Plot the function and its transform and interpret it.

12. A source containing a mixture of hydrogen and deuterium atoms emits a red doublet at $\lambda=6563\,\AA$ whose separation is 1.8Å. Find the minimum number of lines required in a plane transmission grating to resolve the doublet in the second order.

13. a) A beam of light is viewed through a rotating Nicol prism with the beam as axis and no intensity variation is observed. A $\lambda/4$ -plate is placed in front of the Nicol and then it is turned. The light becomes extinct in two positions which differ by 180° . What is the polarization state of the beam of incident light?
b) Plane polarised light of $\lambda=5000\,\AA$ is incident on a quartz plate cut parallel to the optic axis. Find the least thickness of the plate for which the ordinary and extraordinary rays combine to form plane polarised light on emergence. The indices of refraction of quartz are $n_e$=1.5533 and $n_o$=1.5442. (2+3)