



Registration Number:

Date & session:

ST JOSEPH'S UNIVERSITY, BENGALURU-27
M.Sc (MATHEMATICS) - II SEMESTER
SEMESTER EXAMINATION: APRIL, 2023
(Examination conducted in May 2023)
MT 8421 – PARTIAL DIFFERENTIAL EQUATIONS
(For current batch students only)

Time: 2 Hours

Max. Marks: 50

1. This paper contains **ONE** printed page.
2. Attempt any **FIVE FULL** questions.
3. Every question carries **TEN** marks .

1. a) Find the integral surface of the partial differential equation $(x - y)y^2p + (y - x)x^2q = (x^2 + y^2)z$ passing through the curve $xz = a^3, y = 0$.
b) Classify the given partial differential equation and find its characteristics $y^2r - x^2t = 0$. (6+4)
2. Solve $y(x + y)(r - s) - xp - yq - z = 0$ by reducing it to canonical form.
3. a) Solve $(D^2 - 3DD' + 2D'^2)z = e^{2x-y} + e^{x+y}$.
b) Solve $x^2r - 3xys + 2y^2t + px + 2qy = x + 2y$. (5+5)

OR

Solve $(r - s)y + (s - t)x + q - p = 0$ using Monge's method.

4. Solve the Dirichlet problem $\nabla^2 u = 0$, $0 < x < 1$, $0 < y < 1$ subjected to the boundary conditions $u(x, 0) = x(x - 1)$, $u(x, 1) = 0$, $u(0, y) = 0$, $u(1, y) = 0$.
5. Using Riemann-Volterra method, solve $\frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial y^2}$ where $z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ are prescribed along a curve C in the xy -plane.
6. Obtain the general solution of three-dimensional heat equation in cylindrical co-ordinate system.
7. Using the method of eigen function expansion, obtain the solution of

$$\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = x^2 \sin(\pi x),$$

where $0 < x < 1$, $t > 0$ subjected to the conditions

$$\begin{aligned} u(0, t) &= 0, \\ u(1, t) &= 0, \\ u(x, 0) &= \pi, \\ \frac{\partial u}{\partial t}(x, 0) &= 2\pi \sin(2\pi x). \end{aligned}$$