Registration Number:
Date & session:

ST. JOSEPH'S UNIVERSITY, BENGALURU -27
M.Sc. (STATISTICS) – II SEMESTER
SEMESTER EXAMINATION: APRIL 2023
(Examination conducted in May 2023)
ST 8121 – DISTRIBUTION THEORY
(For current batch students only)

Time: 2 Hours **Max Marks: 50**

This paper contains TWO printed pages and ONE part

PART-A

Answer any FIVE of the following **10x5= 50**

1. A) Define Binomial distribution. Obtain the mean and variance of Binomial which is truncated at zero.
   B) State and prove memory less property of exponential distribution.
   C) Using the definition of symmetric random variables show that normal distribution is symmetric. (5+3+2)

2. A) Define Pareto distribution and obtain its mode.
   B) Write the PDF and CDF of Gumbel type 1 bivariate exponential distribution.
   C) For any two random variables X and Y prove that $EE(Y/X) = E(Y)$
   D) If $X \sim N_p(\mu, \Sigma)$, using the MGF of multivariate normal prove that $Y = DX$ follows multivariate normal for any matrix D. (2+2+3+3)

3. A) Find the probability distribution of $\frac{X_1}{X_2}$, where X_1 and X_2 are i.i.d. standard normal.
   B) Suppose that the random sample of size 2, X_1 and X_2 is drawn from exponential with mean 10. If Y_1 and Y_2 be the minimum and maximum determine $P(Y_1 < 5|Y_2 < 10)$.
   C) State Fisher Cochran theorem. (5+3+2)

4. A) If $X_1, \dots, X_n$ be identically independently distributed random variables from a Weibull distribution show that $min\ (X_1, \dots, X_n)$ is also Weibull. (3)
   B) Derive the probability density function of r^{th} order statistic and hence obtain the probability distribution of r^{th} order statistic when observations are from U (0, 1). (7)

5. A) Define non-central chi-square distribution and obtain its MGF. (7)

   B) Write down the probability density function of t distribution. Mention its mean and variance. Justify that standard Cauchy is a special case of t distribution. (3)

6. A) Define order statistic. Write down the distribution function of first and n^{th} order statistic when observations are from iid random variables.

   B) Let $Y_1, Y_2, \ldots Y_n$ be the set of order statistics of independent RVs with common PDF of exponential distribution. Define the normalized spacings $Z_1 = nY_1$, $Z_2 = (n-1)(Y_2 - Y_1)$, $Z_3 = (n-2)(Y_3 - Y_2)$, ... $Z_n = (Y_n - Y_{n-1})$. Prove that $Z_1, Z_2, \ldots, Z_n$ are identically distributed. (3+7)

7. A) If $Y \sim N_p(\mu, \Sigma)$ then prove that $(Y-\mu)'\Sigma^{-1}(Y-\mu) \sim \chi^2(p)$.

   B) Define central F distribution. Mention its mean and variance. (7+3)