ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
M.A. ECONOMICS- I SEMESTER
SEMESTER EXAMINATION: OCTOBER 2019
EC 7418: MATHEMATICAL METHODS FOR ECONOMISTS

This question paper has 3 printed pages and 3 parts

Time: 2.5 Hours   **Maximum Marks-70**

Part A: Answer any 5 of the following questions   **2X5 = 10**

1. Find the market clearing price(p) and quantity (Q) for the following model:
$$Q^d = 3 - p^2$$
$$Q^s = 6p - 4$$
$$Q^d = Q^s$$

2. Given the matrices A, B and C as: A = $\begin{matrix} 2 & 5 \\ 0 & 1 \end{matrix}$, B = $\begin{matrix} 1 & 2 & 1 \\ 2 & 0 & 0 \end{matrix}$ and C = $\begin{matrix} 2 & 2 \\ 0 & 1 \\ 5 & 0 \end{matrix}$
   Calculate A(BC). Is A(BC) same as (AB)C?

3. The functional relationship between demand (y) and price (x) is given by: $y = ax^b$.
   Calculate the price elasticity of demand.

4. The demand function is given by P = 460 − 3Q. Find the consumer's surplus when 92 units of the commodity are sold.

5. Check for Walrasian and Marshallian stability:
   $Q_D = 5 - 2P$
   $Q_S = 1 + 3P$

6. What is a saddle point in the theory of games?

7. Explain the Hawkins-Simon conditions.

Part B. Answer any three of the following: **10 X 3 = 30**

8. Consider the IS-LM model given by the following equation:
$$sY + ar = I^0 + G$$
$$mY - hr = M_s - M^0$$

where s, a, m, h, M_s and M^0 are positive parameters. 'Y' is the net national product; 'r' is the interest rate, 's' the marginal propensity to save, 'a' is the marginal efficiency of capital. Solve for Y and r using Cramer's rule.

9. a) The following are the two demand functions for two commodities X_1 and X_2:
   $X_1 = p_1^{-1.7}p_2^{0.8}$, $X_2 = p_1^{0.5}p_2^{-0.8}$. Determine whether the two commodities are complements or substitutes.
   b) The demand function for a commodity is given by:
   $X_1 = 300 - 0.5p_1^2 + 0.02p_2 + 0.05y$. Find the income elasticity of demand when $p_1 = 12$, $p_2 = 10$ and y (income) $= 200$.

10. Given $q = 75[0.3K^{-0.4} + 0.7L^{-0.4}]^{-2.5}$, find out the degree of homogeneity of this production function and verify Euler's theorem.

11. a) If the production function is given by the equation $q = Ax_1^2x_2^2 - Bx_1^3x_2^3$, show that the equation of expansion path is given by $r_1x_1 - r_2x_2 = 0$, where r_1 and r_2 are the unit prices of x_1 and x_2.
    b) Given the current value of a function $f(t) = P(t)e^{-rt}$ for harvesting a tree, when should the tree be cut down to maximise its PDV (Present Discounted Value)? (Assume that interest rate is r% per year, compounded annually).

12. In a perfectly competitive market demand and supply curves of a commodity are as follows:$P_d = 10 - q$ and $P_s = q + 2$. Find the consumer's surplus and producer's surplus at the equilibrium price.

Part C. Answer any two of the following: 15 X 2 = 30

13. a) An economy produces only coal and steel. The two commodities serve as intermediate inputs in each other's production. 0.4 tonne of steel and 0.7 tonne of coal are needed to produce a tonne of steel. Similarly, 0.1 tonne of steel and 0.6 tonne of coal are required to produce a tonne of coal. No capital inputs are needed. Do you think that the system is viable? Also, 2 and 5 labour days are required to produce a tonne of coal and steel respectively. If the economy needs 100 tonnes of coal and 50 tonnes of steel, calculate the gross output of the two commodities and total labour required.

    b) Suppose the demand and supply set for Mocha-Cola drink are as follows: S consists of pair for which $q - 3p = -5$ while D consists of pairs such that $q + 2p = 145$. Here, p and q denote price and quantity, respectively.
       • Determine the market clearing price and quantity.
       • If government imposes tax of Rs. 10 per bottle, what will be new selling price and quantity sold?

14. a) A manufacturer produces two types of medicines, A and B. The profit per bottle of A and B being Rs. 7 and Rs. 5 respectively. Both A and B require two chemicals C and D. Each bottle of A requires 1 litre of C and 4 litres of D whereas each bottle of B requires 2 litres of C and 3 litres of D. The total supply of C and D are 6 litres and 12 litres respectively. Using Simplex method, find, how many bottles of A and B will the firm produce to maximise its profit?

b) Determine the optimum strategies for the two players X and Y and find the value of the game from the following payoff matrix:

$$
\begin{array}{c}
\text{Player Y} \\
\text{Player X}
\begin{array}{rrrr}
3 & -1 & 4 & 2 \\
-1 & -3 & -7 & 0 \\
4 & -6 & 2 & -9
\end{array}
\end{array}
$$

15. Assume that in a duopoly market, the demand and cost functions of the duopolists (Firm A and B) are:

$P = 100 - 0.5(X_1 + X_2)$

$C_1 = 5X_1$

$C_2 = 0.5X_2{}^2$

a) If firm A is acting as a leader and firm B as a follower, what would be profit maximising quantities for both the firms and also what is the amount of profit for both the firms in this situation?

b) If firm B is now acting as a leader and firm A as a follower, what would be profit maximising quantities for both the firms and also what is the amount of profit for both the firms in this situation?

c) Compare the profit maximising quantities and also the amount of profit in the scenarios,a) and b).