Registration Number:
Date & session:

ST.JOSEPH'S UNIVERSITY, BENGALURU -27
B.Sc. (MATHEMATICS) – I SEMESTER
SEMESTER EXAMINATION: OCTOBER 2023
(Examination conducted in November/ December 2023)
MT 121 – MATHEMATICS- I
(For current students only)

Time: 2 Hours **Max Marks: 60**

This paper contains 2 printed pages and 3 parts

PART A

Answer any Six of the following: **(6X2=12)**

1. Find one eigenvalue of the matrix $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 1 & 4 \\ 1 & 5 & 0 \end{bmatrix}$.
2. Evaluate $D^n(e^x sin^2 x)$.
3. If $y = x^n logx$ then show that $xy_{n+1} = n!$.
4. Give an example of a function $f: [a, b] \to R$ which satisfies intermediate value property but is not continuous on $[a, b]$.
5. State Rolle's theorem.
6. Evaluate $\lim\limits_{x\to 0} \frac{\tan x - x}{x^2 \tan x}$.
7. Find the total differential of $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$.
8. If $u = z - x, v = y - z, w = x + y + z$ find the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

PART B

Answer any three of the following: **(3X6=18)**

9. Find all the eigen values of the matrix $= \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$. Find the set of eigenvectors corresponding to the smallest eigenvalue.

10. (a) Find the general solution of the system

$$x_1 + 3x_2 + x_3 = 0\,, -4x_1 - 9x_2 + 2x_3 = 0, -3x_2 - 6x_3 = 0.$$

(b) Find the value of h such that the matrix $\begin{bmatrix} 2 & -1 & : & 1 \\ 4 & -2 & : & h \end{bmatrix}$ is the augmented matrix of a consistent linear system. (4+2)

11. Define the continuity and differentiability of a function at a point. Also, prove that if a function is differentiable at a point, then it is continuous at that point.

12. If $y = \left(x + \sqrt{x^2 - 1}\right)^m$ then show that

$$(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0.$$

PART C

Answer any five of the following: (5X6=30)

13. State intermediate value theorem. Show that a continuous function $f: [0,1] \to [0,1]$ has a fixed point in [0,1].

14. State and prove Lagrange's mean value theorem.

15. Use Taylor's theorem to prove that $1 + \frac{x}{2} - \frac{x^2}{8} < \sqrt{1 + x} < 1 + \frac{x}{2}$ for all $x > 0$.

16. (a) Let $f: [0,2] \to R$ be a differentiable function such that $f(0) = 0, f(1) = 2, f(2) = 1$. Show that there exists $c \in (0,2)$ such that $f'(c) = 0$.

(b) Show that $u = e^{ax+by} f(ax - by)$ *Prove that* $b\frac{\partial u}{\partial x} + a\frac{\partial u}{\partial y} = 2abu.$ (3+3)

17. If $u = f(r)$ and $r^2 = x^2 + y^2$ show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r).$

18. State and prove Euler's theorem for homogeneous functions and hence prove that if $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0.$

19. Find the maxima and minima of the function $f(x, y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4.$
