Register Number:
Date:

ST JOSEPH'S UNIVERSITY, BENGALURU-27
B.Sc (MATHEMATICS) - V Semester
SEMESTER EXAMINATION: OCTOBER 2023
(Examination conducted in November/December 2023)
MTOE 8 -MATHEMATICS FOR PHYSICAL SCIENCES-III
(Additional OE examination for current V semester students only.)

Duration: 2 Hours **Max. Marks:** 60

This paper contains TWO pages and THREE parts.

PART A

Answer any SIX of the following. **(6×2=12)**

1. Solve $\frac{d^4y}{dx^4} + 2\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} = 0$
2. Solve $\frac{d^2y}{dx^2} + 9y = 0$.
3. Check if the given differential equation is exact, $x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{x^2}$.
4. Find the reciprocal of $z = 2 + 4i$.
5. Give the Cauchy- Riemann equations in polar form.
6. Find the real and imaginary parts of $log(x + iy)$.
7. Find the Laplacian of $\phi(x, y, z) = x^2 - y^2$.
8. Find the value λ such that $\vec{F}(x, y, z) = (3x - 2y + z)\ \hat{i} + (4x + \lambda y - z)\ \hat{j} + (x - y + 2z)\ \hat{k}$ is solenoidal.

PART B

Answer any THREE of the following. **(3×6=18)**

9. Solve $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 9\frac{dy}{dx} - 27y = \cos 3x$.
10. Solve $\frac{d^2y}{dx^2} - 10\frac{dy}{dx} + 16y = e^{4x}\sin 2x$.
11. Solve the simultaneous differential equations, $\frac{dx}{dt} = 3x - y$; $\frac{dy}{dt} = x + y$.
12. Solve the Cauchy Euler differential equation $x^3\frac{d^3y}{dx^3} - 3x\frac{dy}{dx} + 3y = 4x$.
13. Solve by method of variation of parameters, $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{-x}$.

PART C

Answer any <u>FIVE</u> of the following. **(5×6=30)**

14. Check if the function $(x^3 - 3xy^2) + i(3x^2y - y^3)$ is analytic or not.

15. Check if the C-R equations are satisfied for the function $r^2 \cos 2\theta + i\ r^2 \sin 2\theta$.

16. Show that the real and imaginary parts of the function $f(z) = \sin z$ are harmonic.

17. Find the analytic function $f(z) = u + iv$ using Milne Thomson method, where $v = e^{-x}(x \cos y + y \sin y)$.

18. Find the gradient of the scalar point function $\phi(x, y, z) = x^2 + 2y^2 + z$. Also find the directional derivative of ϕ at $(1, 1, 2)$ in the direction of $3\hat{i} - 4\hat{j}$.

19. (i) Prove that $\vec{F} = (x + 3y)\hat{i} + (-y - 2z)\hat{j} + x^2\hat{k}$ is solenoidal.

    (ii) Find the constants a, b, c such that $\vec{F}(x, y, z) = (x + 2y + az)\ \hat{i} + (bx + 3y - z)\ \hat{j} + (4x + cy + 2z)\ \hat{k}$ is irrotational. **(2+4)**

20. Find the divergence and curl of $\vec{F}(x, y, z) = x^2y\ \hat{i} - (z^3 - 3x)\ \hat{j} + 4y^2\ \hat{k}$.