Registration Number:
Date & Session:

ST JOSEPH'S UNIVERSITY, BENGALURU -27
M.Sc (MATHEMATICS) - III SEMESTER
END SEMESTER EXAMINATION : OCTOBER 2023
(Examination conducted in November /December 2023)
MT 9622 : MATHEMATICAL METHODS
(For current batch students only)

Time: 2 Hours **Max Marks: 50**

This paper contains TWO printed pages.

Answer any FIVE full questions. **5 X 10 = 50 Marks**

1. a) Find the resolvent kernel of the Volterra integral equation with kernel $K(x,t)=1.$ [5]

   b) Solve the Fredholm integral equation $u(x)=e^{x}+\lambda\int_{0}^{1}2e^{x}e^{t}\,u(t)\,dt$ using separable kernel method. [5]

2. a) Derive an equivalent integral equation for the initial value problem $y''-3y'+2y=4\sin x$ with $y(0)=1,\ y'(0)=-2.$ [5]

   b) Find the Fourier sine transform of $f(x)=e^{-|x|}$ and hence evaluate $\int_{0}^{\infty}\frac{x\sin mx}{1+x^{2}}dx,\ m>0.$ [5]

3. Find the Fourier transform of $f(x)=\begin{cases}1-|x| & for\ \ |x|\le 1\\ 0 & for\ \ |x|>1\end{cases}$. Hence evaluate $\int_{0}^{\infty}\frac{\sin^{2}y}{y^{2}}dy.$ [10]

OR

Verify convolution theorem for Fourier transform for the functions $f(x)=g(x)=e^{-x^{2}}$. [10]

4. a) Determine the first three terms in the expansion of roots of the equation $x^{2}+2\varepsilon x-3=0$ for small ε. [5]

   b) Define a singular perturbation problem and hence solve $\varepsilon x^{3}-x+1=0.$ [5]

5. Solve the initial value problem $\frac{dy}{dx} + \varepsilon y = x$ with $y(0) = 0$ using perturbation method. [10]

6. a) Find the leading term of the asymptotic expansion for $\int_0^{\infty} e^{-x \sin h^2 t} dt$ as $x \to \infty$. [5]

   b) State Watson's lemma and hence evaluate $\int_0^5 \frac{e^{-xt}}{1+t^2} dt$ as $x \to \infty$. [5]

7. Find the asymptotic expansion for the function $f(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ as $x \to \infty$. [10]