Register Number:

Date and Session:

ST. JOSEPH'S UNIVERSITY, BENGALURU -27
M.Sc (MATHEMATICS) - III SEMESTER
SEMESTER EXAMINATION: OCTOBER 2023
(Examination conducted in November/ December 2023)

MT 9722 - NUMERICAL ANALYSIS

(For current batch students only)

Time: 2 Hours Max Marks: 50

This paper contains **TWO** printed pages.

Answer any FIVE full questions of the following. **[5×10=50]**

1. (a) Solve the system of equation using Thomas algorithm.

$$2x_1 - x_2 = 3$$
$$-x_1 + 2x_2 - x_3 = -3$$
$$-x_2 + 2x_3 = 1$$

(b) Find the first derivative of the function at $x = 1.5$ using the given data:

x	1.5	2.0	2.5	3.0	3.5	4.0
f(x)	3.375	7.0	13.625	24.0	38.875	59.0

[6+4]

2. State and prove Hermite's interpolation formula.

3. (a) Apply Simpson's rule to evaluate the integral $\int_0^1 \int_0^1 \frac{dxdy}{(1+x+y)}$, taking $h = k = 0.5$.

(b) Compute $\int_0^1 \frac{dx}{1+x^2}$ by using Trapezoidal rule, taking $h = 0.5$ and $h = 0.25$. Hence find the value of the given integral using Romberg's method. **[4+6]**

[OR]

(a) Using Adam-Bashforth method, evaluate $y(0.8)$ for the differential equation $y' = x - y^2$ satisfying the data $y(0) = 0, y(0.2) = 0.02, y(0.4) = 0.0795$ and $y(0.6) = 0.1762$.

(b) Determine the values of y at the pivotal points of the interval $(0, 1)$, if y satisfies the boundary value problem $y^{iv} + 81y = 81x^2, y(0) = y(1) = y''(0) = y''(1) = 0$ by taking the step size h=3. **[4+6]**

4. Using fourth order Runge-Kutta method, solve the differential equation $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$ at $x = 0.2$ with the initial conditions as $x = 0, y = 1$ and $y' = 0$.

5. Applying Milne's method, find the solution of $\frac{dy}{dx} = x - y^2$ in the range $0 \leq x \leq 1$ with the boundary condition $y = 0$ at $x = 0$.

6. Solve the Poisson equation $u_{xx} + u_{yy} = -81xy$, where $0 < x < 1$ and $0 < y < 1$. Given that $u(0, y) = 0$; $u(x, 0) = 0$; $u(1, y) = 100$ and $u(x, 1) = 100$ using the step size $h = 1/3$.

7. (a) Using Crank-Nicholson method, obtain the solution of the differential equation

$$\frac{\partial^2 u}{\partial x^2} = 16\frac{\partial u}{\partial t}, \text{ where } 0 < x < 1, t > 0$$

subject to conditions $u(x, 0) = 0, u(0, t) = 0$ and $u(1, t) = 100t$. Compute u for one time step with $h = 0.25$ by Gauss-Seidel iteration method.

(b) Using Schmidt method, obtain the solution of the one-dimensional heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \text{ where } 0 \leq x \leq 1$$

subject to the condition $u(0, t) = u(1, t) = 0$ and $u(x, 0) = sin\pi x$. Carry out computations for two levels taking $h = \frac{1}{3}$ and $k = \frac{1}{36}$. **[5+5]**

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