Register Number: DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. PHYSICS - I SEMESTER

SEMESTER EXAMINATION- OCTOBER 2019

PH 7118 - CLASSICAL MECHANICS

Time-2 1/2 hrs. **Maximum Marks-70**

This question paper has 3 printed pages and 2 parts

PART A

Answer any FIVE full questions. **(5x10=50)**

1. What do we mean by the term "dot cancellation". Illustrate this for two systems.
2. For a Lagrangian that depends on the generalized coordinates q_k, generalized velocities $\dot{q}_k$ and time (explicitly):
   (a) obtain the expression for the energy function h. (**7 Marks**)
   (b) Show that this function is conserved if the Lagrangian is not explicitly dependent on time. (**2 Marks**)
   (c) What is the Jacobi Integral? (**1 Mark**)
3. A particle of reduced mass μ is moving in a central force described by an inverse square law: $F(r)=-\frac{k}{r^2}$, with k being a positive constant (attractive force). Obtain the expression for the radius of closest approach to the central object if the total energy of the particle is positive (greater than zero). How many expressions do you obtain and what are their implications?
4. For a particle moving in a (conservative) central force field, the radial equation of motion works out to be $\mu\ddot{r}-\frac{\ell^2}{\mu r^3}=F(r)$ where μ is the reduced mass, r is the radial coordinate, ℓ is the angular momentum and $F(r)$ the conservative force. From the equation of motion given earlier, show that the total energy of the system is conserved.
5. For each of the n degrees of freedom, the Lagrange equation is given by (where the

subscript i indicates the i^{th} degree of freedom) $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=0$. Using the principles of Legendre Transformations, Transform the Lagrangian $L(q,\dot{q},t)$ to the Hamiltonian $H(q,p,t)$ (while doing the transformation, take care to include the indices as well) and obtain the $2n+1$ relations comprising the Hamilton Equations.

6. To understand continuous systems, it is a usual practice to approximate such systems with a series of discrete systems and then letting the spacing between these discrete system go to zero. As an example, we can represent an infinitely long elastic rod to be composed of an infinite chain of discrete particles of mass m held at a distance a from each other and connected by uniform mass-less springs having force constants k . For such a system, obtain the equation of motion of the i^{th} particle.
7. Consider a system made up of N particles, constrained to positions described by vectors $\vec{\boldsymbol{r}}_i(q_k)$, with q_k being the generalized coordinates corresponding to n degrees of freedom. If a rotation through dq_k is made about some arbitrary axis $\hat{n}$ (as shown in Fig. 1), show that the direction of the generalized force is the same as that of the net torque acting on the system.

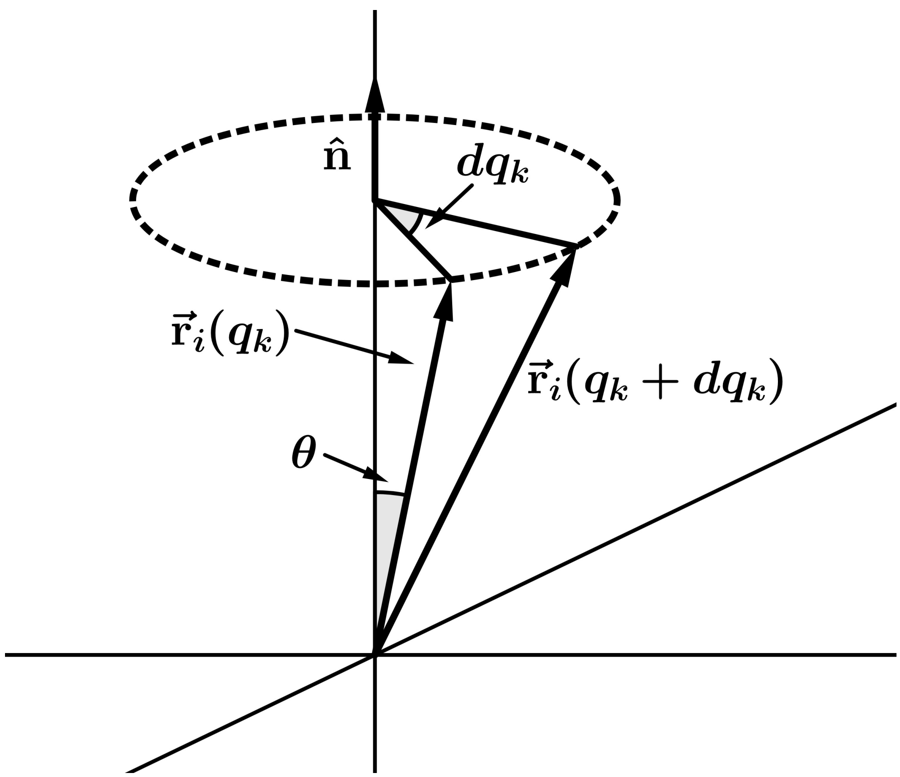

Fig. 1: Rotation of a system.

PART B

Answer any <u>FOUR</u> full questions. **(4x5=20)**

8. Obtain the equation of motion of a bead of mass m constrained to move on a horizontal wire.
9. A stone of mass m is thrown at an angle θ with respect to the horizontal axis. Obtain its equation of motion using the Lagrangian approach.
10. Using the Euler equation, find the extremum of the functional: $J=\int_a^b \left[x+y^2+3\frac{\partial y}{\partial x}\right]dx$
11. For a bead of mass m moving on a horizontal rod, obtain the Hamilton equations of motion.
12. For the central force field (conservative), we have seen that when we define $u=\frac{1}{r}$ the equations of conservation of angular momentum and energy combine to give us a differential equation: $\frac{d^2u}{d\phi^2}+u=-\frac{\mu}{\ell^2}\frac{d}{du}V\left(\frac{1}{u}\right)$ where ϕ is the azimuthal (or angular) coordinate, μ is the reduced mass, ℓ is the angular momentum and $V\left(\frac{1}{u}\right)$ the central potential. If the particle in this potential describes an orbit that is a spiral $r=k\phi^2$, what form would the central potential take?
13. As mentioned in question 6, an elastic rod can be assumed to be a system of an infinite set of mass points connected together by springs. The equation of motion of the i^{th} particle in a system of infinite mass points (with mass m) held together by springs (with force constants k) and with equilibrium separation between the mass points of a is given as: $\frac{m}{a}\ddot{\eta}_i-ka\left(\frac{\eta_{i+1}-\eta_i}{a^2}\right)+ka\left(\frac{\eta_i-\eta_{i-1}}{a^2}\right)=0$. From this, by incorporating the conditions for continuity and that for elasticity of a thin rod, obtain the equation of motion of the elastic rod.