Register Number:
Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. PHYSICS - I SEMESTER

SEMESTER EXAMINATION: OCTOBER 2019

PH 7218 – MATHEMATICAL PHYSICS

Time- $2\frac{1}{2}$hrs **Max Marks-70**

This paper contains THREE printed pages and TWO parts

PART – A

Answer any FIVE. Each question carries 10 marks.

[5 x 10 = 50]

1. Verify the Cayley-Hamilton theorem for the following matrix, $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and use the theorem to find A^{-1}.

2. Show that the function, $f(z) = u + iv$, where $f(z) = \begin{cases} \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}, & z \neq 0 \\ 0 & z = 0 \end{cases}$ satisfies the Cauchy Riemann equation at z = 0. Is the function analytic at z= 0.

3. (a) Evaluate the complex integral $\oint_C \tan z.dz$, where c is $|z| = 2$.

   (b) State and Prove that Cauchy integral theorem. [5+5]

4. Show that

   (a) $x(\pi - x) = \frac{\pi^2}{6} - 4\left[\frac{\cos 2x}{2^2} + \frac{\cos 4x}{4^2} + \frac{\cos 6x}{6^2}\right]$ using half range cosine series for $0 < x < \pi$.

   (b) Using Parseval's formula, show that $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^2}{90}$. [5+5]

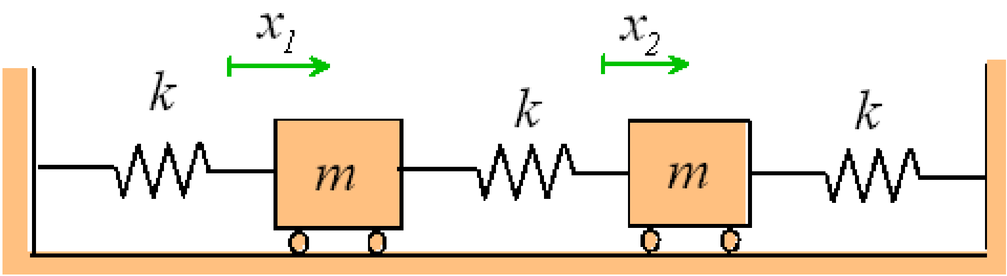
5. Show that,

   $(a)\sqrt{\left(\frac{1}{2}\pi x\right)}\, J_{\frac{3}{2}}(x) = \frac{\sin x}{x} - \cos x,$

   $(b)\sqrt{\left(\frac{1}{2}\pi x\right)}\, J_{\frac{3}{-2}}(x) = -\sin x - \frac{\cos x}{x}$

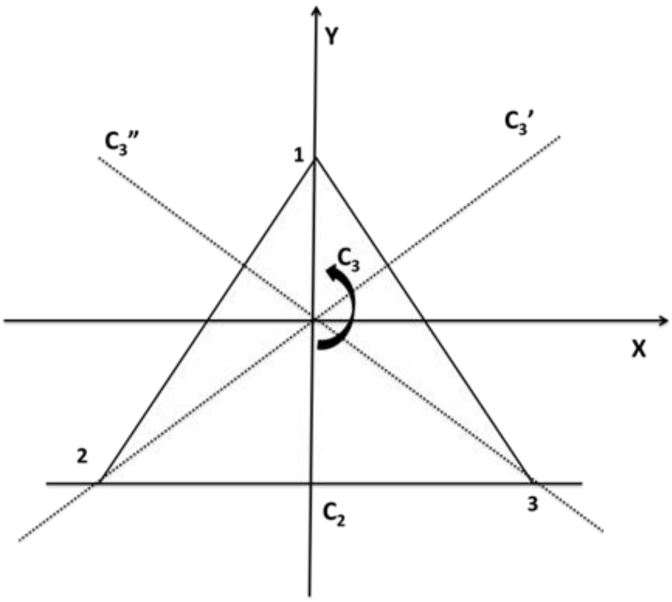
   Where, $J_n(x)$ is the Bessel's Function. [6+4]

6. Write down the equations of two equal masses coupled together and two rigid supports on the sides by identical springs. Find the normal modes of longitudinal oscillations of the system.



7. Find the group elements of an equilateral triangle for six symmetry operations. Mention the condition for each symmetry operation with suitable diagram. Find the Cayley multiplication table of given triangle. Show that $C_2C_3 \neq C_3C_2$.

[2+2+4+2]



PART – B

Answer any FOUR. Each question carries 5 marks.

[4 x 5 = 20]

8. Use Laplace Convolution theorem to find the inverse of the function $\frac{1}{(s^2+a^2)^2}$.

9. Find the Fourier series of the function $f(x) = \begin{cases} -1 \ for & -\pi < x < -\frac{\pi}{2} \\ 0 \ for & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 1 \ for & \frac{\pi}{2} < x < \pi \end{cases}$

10. Prove that $u = x^2 - y^2$ and $v = \frac{y}{x^2+y^2}$ are harmonic functions of (x, y) but are not harmonic conjugates.

11. Find the Fourier transform of the function, $f(x) = \begin{cases} t, & for\ |t| < a \\ 0, & for\ |t| > a \end{cases}$

12. Express f(x) in term of Legendre's polynomials where, $f(x) = 4x^3 - 2x^2 - 3x + 8.$

13. Prove the following Hermite Polynomial relations

$(a)\ H'_{2n+1}(0) = (-1)^n 2^{2n+1} \left(\frac{3}{2}\right)^n.$

$(b)\ H_{2n}(0) = (-1)^n 2^{2n} \left(\frac{1}{2}\right)^n.$

[3+2]