Register Number:
DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

B.Sc. MATHEMATICS– I SEMESTER

SEMESTER EXAMINATION: OCTOBER 2019

MT 118 : MATHEMATICS PAPER I

Time- 2 ½ hrs **Max Marks-70**

This question paper contains FOUR parts and TWO printed pages

I. ANSWER ANY FIVE OF THE FOLLOWING **(5x2=10)**

1. Find the rank of the matrix $A=\begin{bmatrix}1 & 2 & -1 & 4\\ 2 & 4 & 3 & 5\\ 3 & 2 & 6 & 7\end{bmatrix}$ by elementary row operations.
2. Find the value of k for which the following system has a non-trivial solution. $2x-y+2z=0$, $3x+y-z=0$, $kx-2y+z=0$.
3. Find the n^{th} derivative of $y=\cos^2 x$.
4. If $\mathrm{u}=x^2y+y^2z+z^2x$ then show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=(x+y+z)^2$.
5. Evaluate $\int_0^{\pi/4}\tan^4 x\,dx$.
6. Find the point where the line $\frac{x-2}{1}=\frac{y+3}{-1}=\frac{z-1}{-6}$ cuts the plane $2x+y+z=7$.
7. Find the angle between the line and the plane whose direction ratios are $(1,2,-3)$ and $(2,-3,-2)$ respectively.
8. Find the equation of the sphere having the points $(0,1,0)$ and $(3,-5,2)$ at the opposite ends of the diameter.

II. ANSWER ANY THREE OF THE FOLLOWING **(3X6=18)**

9. Reduce the matrix $A=\begin{bmatrix}2 & 3 & 4\\ 5 & 6 & 7\\ 7 & 5 & 6\end{bmatrix}$ to normal form and find its rank.
10. Show that the following equations are consistent and solve, $5x+3y+7z=4$, $3x+26y+2z=9$, $7x+2y+10z=5$.

11. Find the inverse of the matrix $A=\begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$ using elementary transformation.

12. Find a matrix P such that $P^{-1}AP$ is a diagonal matrix, where $A=\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.

III. ANSWER ANY FIVE OF THE FOLLOWING (5x6=30)

13. If $y=\cos(m\cos^{-1}x)$ then prove that $(1-x^2)y_{n+2}-(2n+1)x\,y_{n+1}-(n^2-m^2)y_n=0$

14. If $y=f(r)$ where $r^2=x^2+y^2$ then show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=f''(r)+\frac{1}{r}f'(r)$

15. If u is homogenous function of degree n, then prove that
$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=n(n-1)u$.

16. If $u=f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$ then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0.$

17. If $u=2xy, v=x^2-y^2$, where $x=r\cos\theta, y=r\sin\theta$ then show that $\frac{\partial(u,v)}{\partial(r,\theta)}=-4r^3$.

18. Derive the reduction formula for $\int\cos^n x\,dx$, where $'n'$ is positive integer and hence evaluate $\int_0^{\pi/2}\cos^{10}x\,dx.$

19. Using Leibnitz's rule of differentiation under integral sign, evaluate $\int_0^1\frac{x^\alpha-1}{\log x}dx$ where $\alpha>0$ is a parameter. Hence find $\int_0^1\frac{x^3-1}{\log x}dx$.

IV. ANSWER ANY TWO OF THE FOLLOWING (2x6=12)

20. Find the bisector of the obtuse angle between the planes,
$2x-y+2z+3=0$ and $3x-2y+6z+8=0.$

21. Find the value of the scalar k for which the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{k}$ and $\frac{x-2}{3}=\frac{y-3}{4}=\frac{z-4}{5}$ are coplanar, also find the equation of the plane containing them.

22. Show that $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z=3$ and hence find the point of contact.