



Register No:

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ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE – 27

M.SC. BIG DATA ANALYTICS - I SEMESTER

SEMESTER EXAMINATION – JANUARY 2021

BDA1320: LINEAR ALGEBRA AND LINEAR PROGRAMMING

DURATION: 2.5 HOURS

MAXIMUM MARKS: 70

THIS QUESTION PAPER CONTAINS THREE PRINTED PAGES

STUDENTS ARE ALLOWED TO USE SCIENTIFIC CALCULATORS

ANSWER ANY SEVEN QUESTIONS

Notations:

α	Scalar
v, x, y	Vector
R	Real Line
\in	belongs to

1. (5+5)

- (a) Define a Vector. Describe the two fundamental vectors and their zero vectors.
(b) State the five properties of Vector Operations.

2. (4+4+2)

- (a) Define Vector Spaces. Describe a Vector Space in Geometry (any dimension), state two Spanning Sets for the same. Similarly, describe a Vector Space in R^n (any 'n'), state two Spanning Sets for the same.
(b) Consider any two geometric vectors starting from O, say v_1 & v_2 . Sketch the geometry of

$$\alpha_1 v_1 + \alpha_2 v_2 \text{ where } \alpha_1, \alpha_2 \in R \text{ and } \alpha_1 + \alpha_2 = 1$$

Hint: α_1 & α_2 are scalars, visualize the resultant vector by taking various values of α_1 & α_2 satisfying the condition $\alpha_1 + \alpha_2 = 1$.

- (c) Consider $v = \begin{bmatrix} \alpha \\ 2\alpha \end{bmatrix}$, $\alpha \in R$. Though $v \in R^2$, what is the dimension of v ?

3. (7+3)

(a) Converting a System of Linear Equations to the form: $Ax = y$. What is the significance of Column Space and Null Space of A?

(b) Intuitively with an example in Geometric Vector Space (of a dimension of your choice), argue why Union of any Two Vector Subspaces (from the example Vector Space) is not a Vector Space.

4.

(4+6)

(a) When is a matrix said to be in Row Reduced Echelon Form. Explain its significance.

(b) Solve the following System of Linear Equations using Gaussian Elimination (Use Column and Null Space):

$$4x_1 + 3x_2 + 2x_3 + x_4 = 20$$

$$2x_1 + x_2 + 5x_3 + 3x_4 = 31$$

$$5x_1 - 2x_2 + 4x_3 - x_4 = 9$$

$$x_1 + 2x_2 + 3x_3 + 4x_4 = 30$$

5.

(6+4)

(a) Explain Linear Transformation using two examples – one with a matrix of Linearly Independent Columns and the other with a matrix of Linearly Dependent Columns.

(b) Explain the concept of Basis. Consider the Vector Space R^2 , how many basis vectors are present? Which is the most widely used basis and what is it called?

6.

(6+4)

(a) Explain Eigenvalues and Eigenvectors with an example.

(b) As an application of Eigenvectors, explain the Page Rank Algorithm.

7.

(3+4+5)

(a) Define LPP and write down the general LPP

(b) Define the terms Decision variables, Objective function, linear restriction and non-negative restrictions of an LPP

(c) What do you mean by Basic variables, Non basic variables and basic solution?

8.

(10)

A firm is producing two types of items, say type I and type II. Each unit quantity of type I item requires 5 kgs of raw material and 2 hours of labor time. Each unit quantity of type II item requires 4 kgs of raw material and 8 hours of labor time. The total quantity of raw material available is 20 kgs and the total time available is 16 hours. By selling 1-unit quantity of type I item the firm will get the profit of worth Rs 6 and by selling 1-unit quantity of type II item the firm will get the profit of worth Rs 7. Formulate the LPP and solve using graphical method.

9.

(4+6)

(a) Explain the terms, Unique solution, multiple solution, infeasible solution, and Unbounded solution

(b) Solve the following LPP using Simplex Method:

$$\text{Maximize } Z = 5X + 3Y$$

Subject to

$$3X + 5Y \leq 15$$

$$5X + 2Y \leq 10$$

$$X_1 \geq 0, X_2 \geq 0$$

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