Register Number:

Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
B.Sc MATHEMATICS-III SEMESTER
SEMESTER EXAMINATION: OCTOBER 2019
MT318: MATHEMATICS-III

Time: 2.5 Hours **Max. Marks: 70**

The paper contains Two pages and Four parts .

I. **ANSWER ANY FIVE OF THE FOLLOWING.** $(\mathbf{5 \times 2 = 10})$

1. If a and b are distinct elements of a group G, then prove that either $a^2 \neq b^2$ or $a^3 \neq b^3$.
2. If a is an element of order 8 then find the order of a^4 and a^5 in $< a >$.
3. Prove that the subgroup $SL_2(\mathbb{R})$ is normal in $GL_2(\mathbb{R})$ where $SL_2(\mathbb{R}) = \left\{ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R} \text{ and } \det A = 1 \right\}$ and $GL_2(\mathbb{R}) = \left\{ A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a, b, c, d \in \mathbb{R} \text{ and } \det A \neq 0 \right\}$.
4. Prove that $f : (\mathbb{R}, +) \to (\mathbb{R}^*, \times)$ given by $f(x) = e^x$ is a group homomorphism.
5. Show that $|x|$ is not differentiable at $x = 0$.
6. Examine whether the point $(-2, -1)$ is an extremum of the function $f(x, y) = 2x^2 - xy + y^2 + 7x$.
7. Evaluate $\lim\limits_{x \to 0} \dfrac{\log(\sin x)}{\cot x}$.
8. Solve the differential equation, $(D^2 - 7D + 12)y = 0$ where $D = \dfrac{d}{dx}$.

II. **ANSWER ANY THREE OF THE FOLLOWING.** $(\mathbf{3 \times 6 = 18})$

9. Prove that every subgroup of a cyclic group is cyclic.
10. (a) State and prove Lagrange's theorem for finite groups. [4]
    (b) Without explicitly computing, explain why $\langle 2 \rangle = \langle 26 \rangle$ in $\mathbb{Z}_{30}$. [2]
11. (a) Let H be a subgroup of a group G. Define the normalizer of H in G. [1]
    (b) Let H be a normal subgroup of G and K be any subgroup of G. Show that the set $HK := \{hk : h \in H \text{ and } k \in K\}$, is a subgroup of G. [5]

12. Let $\phi : G \to G'$ be a group homomorphism with Kernel K. Prove the following:
    (a) K is a subgroup of G. [3]
    (b) If H is a cyclic subgroup of G then $\phi(H)$ is a cyclic subgroup of G'. [3]

13. State and prove fundamental theorem of homomorphism for groups.

III. **ANSWER ANY FOUR OF THE FOLLOWING.** $(\mathbf{4 \times 6 = 24})$

14. Prove that a function which is continuous in a closed interval attains its bounds.

15. State and prove Rolle's Theorem.

16. Expand the function $\log(\sec x)$ upto the term containing x^6 by Maclaurin's expansion.

17. Using the method of Lagrange's undetermined multipliers, find the maximum volume of a rectangular box with given surface area.

18. Evaluate $\lim_{x \to 0} \left(\frac{1}{x}\right)^{\tan x}$.

IV. **ANSWER ANY THREE OF THE FOLLOWING.** $(\mathbf{3 \times 6 = 18})$

19. Solve the differential equation, $y'' - 3y' + 2y = xe^x$.

20. Solve the differential equation, $x\frac{d^2y}{dx^2} - 2(1+x)\frac{dy}{dx} + (x+2)y = (x-2)e^x$ where $x > 0$ given that $u = e^x$ is a part of the complementary function.

21. Solve the differential equation, $\frac{d^2y}{dx^2} - \frac{1}{x}\frac{dy}{dx} + 4x^2y = x^4$ by changing the independent variable.

22. Solve the differential equation, $x^2y'' + xy' - y = x^2e^x$ by method of Variation of Parameters.