Register Number:
Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
B.Sc. MATHEMATICS-III SEMESTER
SEMESTER EXAMINATION: OCTOBER-2019
MT-318: MATHEMATICS III

Duration: 2.5 Hours **Max. Marks: 70**

The paper contains TWO pages and FOUR parts

I. ANSWER ANY FIVE OF THE FOLLOWING. (5x2=10)

1. Find two generators of $(\mathbb{Z}_8, +_8)$.
2. Write all distinct cosets of $H = (4\mathbb{Z}, +)$ in a group $G = (\mathbb{Z}, +)$.
3. State Lagrange's Theorem for finite Groups.
4. Let $G = (\mathbb{R}, +)$ be a group of Real numbers and $f : G \to G$ be a mapping defined by $f(x) = 3x$. Is f a homomorphism? Justify.
5. Check whether the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \frac{1}{x}$ is continuous at the point $x = 0$.
6. State Rolle's Theorem.
7. Evaluate $\lim_{x\to 0} \dfrac{1 - cosx}{x^2}$
8. Solve the differential equation $y'' - 2y' + y = 0$.

II. ANSWER ANY THREE OF THE FOLLOWING. (3x6=18)

9. a. Define an index of a subgroup of a finite group and find the index of $H = \{0, 2, 4\}$ which is a subgroup of $G = (\mathbb{Z}_6, +_6)$.
   b. Let H be a subgroup of a group G. Let $a, b \in G$. then prove that $aH = bH$ if and only if $a \in bH$.
   [2+4]
10. If H is a normal subgroup in a group G and K is a subgroup of G, then prove that HK is a subgroup in G.
11. Let G be a group and let $Z(G)$ be the center of a group G. If $G/Z(G)$ is cyclic, then prove that G is abelian.
12. a. Define group homomorphism.
    b. Let $\phi : G \to G'$ be a group homomorphism. If H is normal subgroup in G, then prove that $\phi(H)$ is a normal subgroup in G'. [1+5]
13. State and prove Fundamental theorem of Homomorphism of Groups.

III. ANSWER ANY FOUR OF THE FOLLOWING **(4x6=24)**

14. Prove that every continuous function defined on a closed interval is bounded.

15. a. Examine the differentiability at $x = 0$ of the function $f(x)$ defined by
$$f(x) = \begin{cases} 1+2x & \text{if } \; -1 \leq x \leq 0 \\ 1-3x & \text{if } \; 0 < x \leq 1 \end{cases}$$
    b. Verify the Rolle's theorem for the function $f(x) = x^2 - 6x + 8$ in the interval $[2, 4]$ [3+3]

16. State and prove Cauchy's Mean Value theorem.

17. Find the extreme value for the function $x^3y^2(12 - x - y)$ with $x > 0, y > 0$.

18. Evaluate $\lim_{x \to 0} \left[\frac{1}{x^2} - \frac{1}{xtanx} \right]$.

IV. ANSWER ANY THREE OF THE FOLLOWING. **(3x6=18)**

19. Solve the differential equation $y'' + 3y' + 2y = e^{2x} sin(x)$.

20. Solve the differential equation $4x^2y'' + 4xy' - y = 4x^2$.

21. Solve the differential equation $y'' - y = \frac{2}{1+e^x}$ by the method of variation of parameter.

22. Verify the exactness and solve the differential equation $x^2(1+x)y'' + 2x(2+3x)y' + 2(1+3x)y = 0$.