Register Number:
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ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. PHYSICS - I SEMESTER

SEMESTER EXAMINATION (SUPPLEMENTARY) - OCTOBER 2018

PH 7118 - CLASSICAL MECHANICS

Time-2 1/2 hrs. **Maximum Marks-70**

This question paper has 3 printed pages and 2 parts

PART A

Answer any FIVE full questions. **(5x10=50)**

1. If L is the Lagrangian for a system having n degrees of freedom and satisfying the Lagrange's equation of motion: $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right)-\frac{\partial L}{\partial q_k}=0$, show that changing the Lagrangian into the form: $L^{'}=L+\frac{dF}{dt}$ (where $F=F(q_{1,}q_{2,}q_{3,}\ldots,q_k,t)$ is an arbitrary function that is differentiable) keeps the Lagrange's equation invariant.
2. A system described by a set of generalized coordinates q_k undergoes a change dq_k due to translation.
   (a) Show that the generalized force $Q_k=\sum_{i=1}^{N}\vec{\boldsymbol{F}}_{\boldsymbol{i}}\cdot\frac{\partial \vec{\boldsymbol{r}}_{\boldsymbol{i}}}{\partial q_k}$ is equal to the net force along the direction of translation. (**3 Marks**)
   (b) Show also that the generalized momentum p_k is equal to the net linear momentum along the direction of translation (**3 Marks**)
   (c) If q_k is cyclic, what does this imply for the generalized forces Q_k ? What does it imply for the linear momentum? (**4 Marks**)
3. Write down the Lagrangian for a particle of mass m moving in a central force field potential $V(r)$ (that is conservative and dependent only on the radial component r of the position of the particle)
   (a) What will be the coordinate system to be used for the generalized coordinates. What are the symmetries in the problem and what are the conserved quantities? (**2 Marks**)
   (b) Write down the Lagrangian of the system in terms of the first integrals. (**2 Marks**)
   (c) What are the differential equations, the solutions of which will provide us the position of the

particle? (**6 Marks**)

4. Starting with the definition $\mathcal{F}=\sum_{i=1}^{N}\vec{\boldsymbol{p}}_i\cdot\vec{\boldsymbol{r}}_i$ for N particles in a central force field, obtain the relation for Virial of Clausius (i.e. $\overline{\sum_{i=1}^{N}\vec{\boldsymbol{F}}_i\cdot\vec{\boldsymbol{r}}_i}$)

5. The Poisson Bracket of two functions (of the canonical variables q and p): $f(q,p)$ and $g(q,p)$ is defined as: $[f,g]_{q,p}=\frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i}-\frac{\partial g}{\partial q_i}\frac{\partial f}{\partial p_i}$. Compute
   (a) $[q_i,q_j]_{q,p}$ (**2 Marks**)
   (b) $[p_i,p_j]_{q,p}$ (**2 Marks**)
   (c) $[q_i,p_j]_{q,p}+[p_j,q_i]_{q,p}$ (**2 Marks**)
   (d) $[f,g]_{q,p}+[g,f]_{q,p}$ (**2 Marks**)
   (e) $[f,f]_{q,p}$ (**2 Marks**)

6. Considering a canonical transformation from a set of generalized coordinates and momenta: (q,p) at time t to a new set of *constant quantities* (which may be the $2n$ set of initial values $(q_{0,}p_0)$ at $t=0$, obtain the Hamilton-Jacobi equation (the new Hamiltonian will be related to the old Hamiltonian and the generating function via the equation: $G=H+\frac{\partial F}{\partial t}$.

7. For a rotating rigid body (with an angular velocity $\boldsymbol{\omega}$), it can be shown that any vector $\vec{\boldsymbol{A}}$ representing a point in its interior measured with respect to its center of mass (or origin of the axis of rotation), transforms to inertial frame as: $\left(\frac{d\vec{\boldsymbol{A}}}{dt}\right)_{inertial}=\left(\frac{d\vec{\boldsymbol{A}}}{dt}\right)_{rot}+\vec{\boldsymbol{\omega}}\times\vec{\boldsymbol{A}}$, i.e. we can conceive of a new operator $\left(\frac{d}{dt}\right)_{inertial}=\left(\frac{d}{dt}\right)_{rot}+\vec{\boldsymbol{\omega}}\times$.
   (a) Apply this operator on the position vector $\vec{\boldsymbol{r}}$ to obtain the transformation rule for velocity $\vec{\boldsymbol{v}}$ (explain each term) (**4 Marks**)
   (b) Apply it once more on the velocity vector to obtain the transformation rule for acceleration $\vec{\boldsymbol{a}}$. What is the physical significance of each term you get in this expression? (**6 Marks**)

<u>PART B</u>

Answer any <u>FOUR</u> full questions. **<u>(4x5=20)</u>**

8.
   (a) Write down the Lagrangian of a block of mass m sliding down an inclined plane of angle α (angle between the base of inclined plane and the sloping side). Show your method of working for the computation of the potential energy (**3 Marks**)
   (b) Obtain the equation of motion using Lagrange's equation for this block as it slides down under gravity. (**2 Marks**)

9. Using the Euler equation find the extremum of the following functional: $J=\int_a^b \left(3x+\sqrt{\frac{\partial y}{\partial x}}\right)dx$.

10. The semi-major axis of Neptune's orbit around the Sun is 4.495×10^{12} m . With the solar mass being: 1.99×10^{30} kg and the gravitation constant being: 6.674×10^{-11} m^3 kg^{-1} s^{-2} . Using Kepler's Third law compute the period of revolution of Neptune around the Sun in Earth Years.

11. The Lagrangian for the Simple Harmonic Oscillator is given as: $L=\frac{1}{2}m\dot{x}^2-\frac{1}{2}m\omega^2x^2$ where x is the generalized coordinate and $\dot{x}$ is the generalized velocity. Compute the Hamiltonian of this system.

12. The Hamiltonian for a Simple Harmonic Oscillator is given as: $H=\frac{p^2}{2m}+\frac{1}{2}kq^2$. Obtain the Hamilton Jacobi equation for this system.

13. A bead of mass m is constrained to move in a horizontal circle (the axis of the circle is along the $z-$axis) in the $x-y$ plane on a table.
   (a) Write down the Lagrangian for the bead. (**2 Marks**)
   (b) What is the equation of motion of the block? (**3 Marks**)