Register Number:

Date: 13-01-2020

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
M.Sc. MATHEMATICS - I SEMESTER
SEMESTER EXAMINATION: JANUARY 2021
MT 7518 – DISCRETE MATHEMATICS

Time- 2 ½ HOUR. Max Marks-70

This paper contains TWO printed pages.

Answer any SEVEN of the following. $7 \times 10 = 70$

1. (a) Show that the premises "If you send me an e-mail message, then I will finish writing the program," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed."
(b) Show that the premises "A student in this class has not read the book," and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book." (4+6)

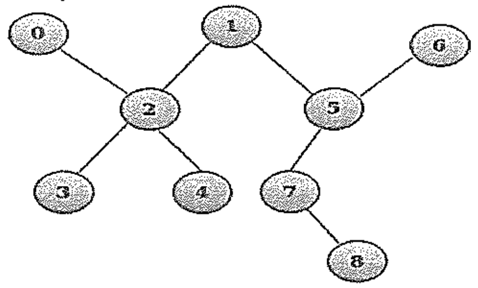
2. (a) Give a proof by contradiction of the theorem "If 3n + 2 is odd, then n is odd."
(b) How many license plates can be made using two letters followed by 4 digits
(i) if repetition of digits and letters are allowed, (ii) if repetition of digits and letters are not allowed (iii) if repetition of digits and letters are allowed and have only vowels and even digits.
(c) Determine the number of integers where $1 \le n \le 100$ and n is not divisible by 2,3 or 5. (3+4+3)

3. (a) Find the solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2$ and $a_1 = 7$.
(b) Find all the solutions to $a_n = 5a_{n-1} - 6a_{n-2} + 7^n$ (5+5)

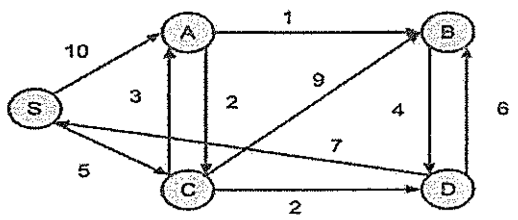
4. (a) Find the domain, range, matrix relation and the digraph for
$A = \{1,2,3,4,8,10\}, B = \{1,4,6,9,20\}$ with $R = \{aRb \; iff \; a/b\}$
(b) Let $A = \{1,2,3,4\}$ and $R = \{(1,2),(2,1),(2,3),(3,4)\}$ Find the relative matrix of transitive closure using Warshall's algorithm. (5+5)

5. (a) Define partial order relation and poset. Show that D_{30} (the set of all positive divisors of 30) is a poset under the condition of division.
(b) Define maximal and minimal elements. Find the maximal and minimal elements of the poset $A = (\{2,4,5,10,12,20,25\}, /)$ by constructing Hasse diagram. (5+5)

6. (a) Define vertex connectivity and edge connectivity of a graph. Find the eccentricity of every vertex and center of the following graph.



(b) Explain Dijkstra's shortest path algorithm. Find the shortest path from source S to all other vertices in the following digraph using the same algorithm.



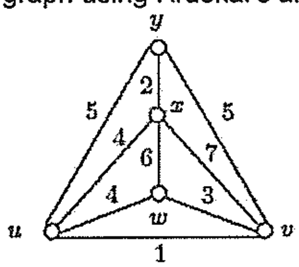
(5+5)

7. Let G be a graph of order $n \geq 3$. If $\deg u + \deg v \geq n$ for each pair u, v of non adjacent vertices of G, then prove that G is Hamiltonian.

8. (a) State and prove Euler's theorem.
   (b) If $G(p,q)$ is a maximal outer planar graph with $p \geq 3$ vertices all lying on the exterior face, then prove that G has $p-2$ interior faces.
   (c) A connected k-regular graph of order 12 is embedded in the plane, resulting in 8 regions. What is k? (5+3+2)

9. For every graph G of order n, having no isolated vertices, prove that $\alpha_0 + \beta_0 = n = \alpha_1 + \beta_1$ where α_0 is the vertex covering number, α_1 is the edge covering number, β_0 is the vertex independence number and , β_1 is the edge independence number of G.

10. (a) Prove that every nontrivial tree has at least two end vertices.
    (b) Explain Kruskal's algorithm. Find the minimum spanning tree of the following graph using Kruskal's algorithm. (5+5)



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