Register Number:
DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. PHYSICS - II SEMESTER

SEMESTER EXAMINATION: APRIL 2019

PH 8418 - QUANTUM MECHANICS - I

Time- 2 1/2 hrs. **Max Marks-70**

This question paper has 3 printed pages and 2 parts

PART A

Answer any FIVE full questions. **(5x10=50)**

1.
(a) How does Young's Double Slit experiment for light depart from classical expectations? Explain the differences using sketches of what you would expect classically.
(b) Explain the degeneracy of energy states with particle in a two dimensional box as an example. **(5+5)**

2.
(a) Using separation of variables method, obtain the expression for Azimuthal equation and discuss the solution for it in the case of spherically symmetric particle.
(b) Show that $\frac{d\langle p_x\rangle}{dt} = \left\langle -\frac{dV}{dx}\right\rangle$, (where p_x is the momentum operator) using Heisenberg representation. **(5+5)**

3.
(a) What are operators? Mention the properties of Hermitian operators. Explain their significance in quantum mechanics.
(b) Can the operators L^2 and L_i; i=x,y,z have simultaneous eigenfunctions? Explain.
(c) if $[L_i, L_j] \neq 0$, explain the physical significance of that. **(4+4+2)**

4. With a and $a^\dagger$ as ladder operators with $a = \left(\frac{m\omega}{2\hbar}\right)^{\frac{1}{2}} x + i\left(\frac{1}{2m\hbar\omega}\right)^{\frac{1}{2}} p$, where x

and p are position and momentum operators, obtain the energy eigen value and normalized wave function for the ground state of a linear harmonic oscillator.

5. Sketch the step potential of the form: $V(x)=\begin{cases} V_0 & \text{for } -\infty<x\le 0 \\ 0 & \text{for } 0<x<\infty \end{cases}$. What are the different regions in this potential and what will be the solution of Schrodinger equation in these regions for a particle having a total energy $E<V_0$? What are the conditions the wavefunction needs to satisfy at the boundaries of the various regions?

6. For a particle in an infinite potential, what is the uncertainty in the momentum of the particle in the ground state?

7. Write down the time independent Schrodinger equation for a free particle ($V=0$) in one dimension. What is the wavefunction that is a solution to this equation? How will you make the wavefunctions normalizable? Once normalized, how will the coefficients of the wavefunction be described?

PART B

Answer any <u>FOUR</u> full questions. (**<u>4x5=20</u>**)

[Constants: **h**=6.626070x10^{-34} J s (**Planck's constant**), **1eV** = 1.6x10^{-19} J (**electron volt to Joules**), **c**=2.99792458x10^{8} m/s (**speed of light**), **1Å** = 1x10^{-10}m (**Angstrom to meters**), **e** = 1.602176x10^{-19} C (**electronic charge**), $\mathbf{m}_{\mathbf{electron}}$=9.10938356x$10^{-31}$kg (**mass of electron**)]

8. A Compton scattering experiment has a UV laser gun encased in Aluminium. Aluminium has a 12th Ionization potential (treat this as work function) of $2085.98\ \text{eV}$. A photon of wavelength $5.904\ \mathrm{A}^\circ$ strikes a free electron and gets scattered (**at an obtuse angle**) and hits the aluminium casing causing photoelectric effect. What is the angle of scattering if the electrons created via photoelectric effect are at rest? Does your result verify that the angle is indeed obtuse?

9. A particle with total energy $E=E_0$ at time $t=0$ units has a wavefunction $\Psi(x,t=0)=\sqrt{\frac{2}{5}}e^{\left(-\frac{x}{5}\right)}$ in the domain $(0,\infty)$ (where x has the dimensions of length). What is the wavefunction of the particle at time $t=5$ units ?

10. Show that $\psi(x)=e^{-\frac{x^2}{2}}$ is an eigen function of the operator $\left(\frac{d^2}{dx^2}-x^2\right)$. Find the corresponding eigen value.

11. A particle on the x-axis has the wave function $\psi(x)=cx^2$ between $x=0$ and $x=2$
    (a) Normalize the wave function over the interval.
    (b) Find the expectation value of the particle's position x .

12. If an atom with one electron having a radius of $0.5\ \mathrm{A}^\circ$ is modeled as an electron in an infinite potential well, what is the work function (i.e. the ionization potential or energy corresponding to the ground state) for this atom?

13. Consider a physical system whose Hamiltonian H and initial state are given by:

$$H = \epsilon \begin{pmatrix} 0 & i & 0 \\ -i & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \text{ and } |\psi_0\rangle = \frac{1}{\sqrt{5}} \begin{pmatrix} 1-i \\ 1-i \\ 1 \end{pmatrix}$$, where ϵ has dimensions of energy.

(a) If we measure the energy, what values will be obtain?

(b) Calculate the expecation value of the Hamiltonian. (**3+2**)