Date:
Reg.No.

St. Joseph's College (Autonomous) Bangalore
VI Semester Examination April-2019
B.Sc Mathematics
MT 6115 - Mathematics VII

Time: $2\frac{1}{2}$ hrs. **Marks: 70**

This question paper has Three parts and One printed page.

I Answer any five questions: (5x2=10)

1. For which value of k will the vector $(1,-2,k)$ in R^3 be a linear combination of the vectors $(3,0,-2)$ and $(2,-1,-5)$.
2. Find the basis of the subspace spanned by the vectors $(2,4,2),(1,-1,0),(1,2,1),(0,3,1)$.
3. Find the linear transformation $f: R^2 \to R^2$ such that $f(1,1)=(0,1)$ and $f(-1,1)=(3,2)$.
4. Find the expression for elementary arc-length and volume element for Cartesian coordinates.
5. Show that the scalar factors for cylindrical polar coordinates are $1, \rho, 1$.
6. Form a partial differential equation by eliminating the arbitrary function f from the equation $z=f(x^2+y^2)$.
7. Solve $(p+q)(z-xp-yq)=1$
8. Solve $(y-z)p+(z-x)q=x-y$

II Answer any three questions: (3x6=18)

9. Show that the union of two subspaces of a vector space Vover a field F is a subspace if and only if one is contained in the other.
10. Given the matrix $A=\begin{pmatrix}1 & -1 & 2\\ 3 & 1 & 0\end{pmatrix}$ determine the linear transformation $T: V_3(R) \to V_2(R)$ relative to bases $B_1=\{(1,1,1),(1,2,3),(1,0,0)\}$; $B_2=\{(1,1),(1,-1)\}$
11. If T is a linear transformation from V_3(R) into $V_4(R)$ defined by
 $T(1,0,0)=(0,1,0,2)$, $T(0,1,0)=(0,1,1,0)$, $T(0,0,1)=(0,1,-1,4)$
 Find the range, null space, rank and nullity of T.
12. Show that every vector space V over the real field R of dimension n is isomorphic to $V_n(R)$.

III Answer any seven questions: (7x6=42)

13. Show that the spherical coordinate system is an orthogonal curvilinear coordinate system.
14. Derive the unit vectors e_ρ, e_φ, e_z in terms of i,j,k hence find i,j,k in terms of e_ρ, e_φ, e_z.
15. Express the vector $f=zi-2xj+yk$ in terms of cylindrical coordinates.
16. Verify the condition for integrability and solve $(2x^2+2xy+2xz^2+1)dx+dy+2zdz=0$
17. Solve $x^2p^2+y^2q^2=z^2$ using the transformation $logx=u, logy=v, logz=w$
18. Solve $p(1+q)=zq$
19. Find the complete integral of $(p^2+q^2)y=qz$ by Charpit's method.
20. Solve $2r-s-3t=e^{x-y}$
21. Derive the Fourier series solution of the one dimensional heat equation.
