



Register Number:

DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
B.Sc. MATHEMATICS – VI SEMESTER
SEMESTER EXAMINATION: APRIL 2019
MT6215 : MATHEMATICS-VIII

Time- 2 ½ hrs

Max Marks-70

This question paper has **two** printed pages and **three** parts.

I. ANSWER ANY FIVE OF THE FOLLOWING

(5X2=10)

1) Show that $|z+1|^2 + |z-1|^2 = 4$ represents a circle.

2) Find $\lim_{z \rightarrow 1+i} \left(\frac{z^2 - z + 1 - i}{z^2 - 2z + 2} \right)$

3) Show that $u = x^3 - 3xy^2$ is harmonic.

4) Evaluate $\int_0^{1+i} (x^2 - iy) dz$ along $y = x$.

5) Evaluate $\int_C \frac{e^z}{z^3}$, where C is the circle $|z| = 1$.

6) Find the Laplace transform of $t^3 + 3 + \sin 3t$.

7) Deduce the Laplace transform of e^{kt} .

8) Find the inverse Laplace transform of $\frac{s+1}{s^2 + 2s - 8}$.

II. ANSWER ANY SEVEN OF THE FOLLOWING

(7X6=42)

9) Define Analytic function. State and prove the necessary condition for a function $f(z) = u + iv$ to be analytic.

10) a) Show that $\arg\left(\frac{z-1+i}{z+i}\right) = \frac{\pi}{4}$ represents a circle. Find its centre and radius.

b) Show that the function $f(z) = (x^2 + y^2) + i2xy$ is not differentiable $\forall z \neq 0$. **(4+2)**

11) a) Show that $f(z) = \cos z$ is analytic and $f'(z) = -\sin z$.

b) Show that $f(z) = e^y (\cos x + i \sin x)$ is not analytic. **(4+2)**

12) Find the Orthogonal trajectories of the following family of curves:

$$e^{-x}(x \sin y - y \cos y) = c.$$

13) Find the analytic function whose real part is $r^2 \cos 2\theta - r \sin \theta$ and hence find its imaginary part.

14) State and Prove Cauchy's Integral Formula.

15) Evaluate: $\int_C \frac{2\pi z}{(z^2 + 1)^2} dz$, where C is the square with vertices $\pm 2 \pm 2i$.

16) Discuss the transformation $w = \sin z$.

17) a) Show that the transformation $w = \frac{4z - 5}{2 - 4z}$ maps the circle $|z| = 1$ onto a circle of radius unity in the w -plane.

b) Find the fixed point of the transformation. $w = \frac{z - 1}{z + 1}$. **(4+2)**

III. ANSWER ANY THREE OF THE FOLLOWING

(3X6=18)

18) Using Regula- Falsi method find a root of the equation $x^{2.2} = 69$ in the interval (5, 8), correct to 3 decimal places.

19) a) Find the approximate solution at $x = 1.2$ of the equation $\frac{dy}{dx} = xy$ given $y(1) = 2$ by

Runge- Kutta method of 4th order.

b) Mention the formula for Newton Raphson method. **(5+1)**

20) Show that $L[f(t)] = \frac{\omega}{(s^2 + \omega^2)(e^{\frac{\pi s}{\omega}} + 1)}$, where

$$f(t) = \begin{cases} 0, & 0 < t < \frac{\pi}{\omega} \\ -\sin \omega t, & \frac{\pi}{\omega} < t < \frac{2\pi}{\omega} \end{cases} \text{ and } f\left(t + \frac{2\pi}{\omega}\right) = f(t).$$

21) a) Find the Laplace transform of $f(t) = \sin t + 5 \int_0^t f(u) \sin(t - u) du$.

b) Find the Laplace transform of $f(t) = \frac{\sin ht}{t}$. **(3+3)**

