Register Number:

Date:

St. Joseph's College (Autonomous), Bangalore
B.Sc Mathematics-IV Semester
Semester Examination: April, 2019
MT-415: Mathematics-IV

Time: $1\frac{1}{2}$ Hours **Max. Marks:** 35

The paper contains one page

ANSWER ANY SEVEN OF THE FOLLOWING QUESTIONS 7*5=35

1. If N is a normal subgroup of G and H is a subgroup of G, then prove that NH is a subgroup of G.
2. If N is a normal subgroup of G and a is an element of finite order in G, then show that the element Na of the quotient group $G|N$ is also of finite order and order of Na divides the order of a.
3. If $f : G \to G$ be a homomorphism from the group G into itself and H is a cyclic subgroup of G, then show that f(H) is again cyclic subgroup of G.
4. If $f : G \to G'$ be an isomorphism of a group G onto a group G' and a is any element of G, then prove that the order of f(a) equals the order of a.
5. Find the fourier expansion of the function $f(x) = \begin{cases} -1 & -3 < x < 0 \\ 0 & x = 0 \\ 1 & 0 < x < 3 \end{cases}$
6. Obtain the fourier series of $f(x) = \begin{cases} x & -\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2} \\ \pi - x & \frac{\pi}{2} \leqslant x \leqslant \frac{3\pi}{2} \end{cases}$
7. Find the fourier cosine series for $f(x) = x, 0 \leqslant x \leqslant L$.
8. Find the Taylor polynomial of $f(x, y) = \log(1 + x + y)$ at $x = 0, y = 0$.
9. Show that a rectangular box of maximum volume with prescribed surface area is a cube.
10. Find the three numbers x,y,z such that $x + y + z = 1$ and xy+yz+zx is maximum.