The paper contains <u>Two</u> pages and <u>Four</u> parts .

I. <u>**ANSWER ANY FIVE OF THE FOLLOWING.**</u> $(5 \times 2 = 10)$

1. Define a group.

2. Show that the identity element of a group is unique.

3. Draw the Cayley's table for the set $\{5,15,25,35\}$ under multiplication modulo 40. Find the identity element.

4. Find the truth set of the open sentence $p(x) : |x - 1| \leq 2$ where R[p(x)]=$\mathbb{N}$, the set of natural numbers.

5. Find $\frac{ds}{dx}$ and $\frac{ds}{dy}$ for the curve $y = a \cosh(\frac{x}{a})$.

6. Find the asymptotes parallel to the coordinate axes for the curve $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$.

7. Find the area bounded by the curve $a^2 y = x^2(x + a)$ and the x-axis.

8. By grouping the terms of the equation solve $ydx - xdy + 2xy^2 dx - 2y^3 dy = 0$.

II. <u>**ANSWER ANY TWO OF THE FOLLOWING.**</u> $(2 \times 6 = 12)$

9. Show that the set $\{\begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \in \mathbb{R}, a \neq 0\}$ is a group with respect to matrix multiplication.

10. (a) Prove that the intersction of any two subgroups of a group G is also a subgroup.   [3]
    (b) Let G be an abelian group. Prove that $H = \{x^2 : x \in G\}$ is a subgroup of G.   [3]

11. If $p(x)$ and $q(x)$ are two open sentences with same replacement set then show that
    (a) $T[p(x) \wedge q(x)] = T[p(x)] \bigcap T[q(x)]$ [3]
    (b) $T[p(x) \vee q(x)] = T[p(x)] \bigcup T[q(x)]$ [3]

III. **ANSWER ANY FIVE OF THE FOLLOWING.** $(5 \times 6 = 30)$

12. (a) Prove that $\tan \phi = r\frac{d\theta}{dr}$ for the polar curve $r = f(\theta)$. [4]

    (b) Find the angle between the radius vector and the tangent vector for the curve
    $r = a(1 + \sin\theta)$ at $\theta = \frac{\pi}{6}$. [2]

13. Show that the pedal equation of the curve $x = a\cos^3 t$ and $y = a\sin^3 t$ is $r^2 = a^2 - 3p^2$.

14. Show that the radius of curvature of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\rho = \frac{a^2 b^2}{p^3}$.

15. Find all the asymptotes of the curve $4x^2(y - x) + y(y - 2)(x - y) = 4x + 4y - 7$.

16. Find the position and nature of the double points of the curve $x^3 - y^2 - 7x^2 + 4y + 15x - 13 = 0$.

17. Find the length of an arc of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$.

18. Find the volume of the solid obtained by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line.

IV. **ANSWER ANY THREE OF THE FOLLOWING.** $(3 \times 6 = 18)$

19. Solve $x\frac{dy}{dx} + y\log y = xye^x$.

20. Find the suitable integrating factor and solve $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$.

21. Find the general and singular solution of $(p - 1)e^{3x} + p^3 e^{2y} = 0$ using the substitution $u = e^x$ and $v = e^y$.

22. Find the orthogonal trajectory of the family of circles through the origin and having the centres on the x-axis.