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Register Number:

DATE:

**ST.JOSEPH’S COLLEGE (AUTONOMOUS), BANGALORE- 27**

**M.Sc. MATHEMATICS- IV SEMESTER**

**SEMESTER EXAMINATION- APRIL 2018**

**MT 0114: Measure and Integration**

**Time:** 2 ½ **Hours Max. Marks: 70**

**This question paper has two printed pages.**

**Answer Any SEVEN Questions**1**.** i) Define an outer measure. (1)
 ii) Prove that outer measure is monotone (2)
 iii) Prove that the outer measure is translation invariant. (3)
 iv) Prove that the outer measure is countably sub additive. (4)

2. i) Define a measurable set. (1)
 ii) Prove that any set of outer measure zero is measurable. (2)
 iii) Prove that the interval ( a ,  ) is measurable. (7)

3. i) Let A be any set and $\{E\_{k}\}\_{k=1}^{n}$ be a finite , disjoint collection of measurabe sets. Then
 prove that m\*$\left(A∩\left[\bigcup\_{k=1}^{n}E\_{k}\right]\right)$ = $\sum\_{k=1}^{n}m^{\*}(A∩E\_{k})$
 and in particular $m^{\*}\left(\bigcup\_{k=1}^{n}E\_{k}\right)=\sum\_{k=1}^{n}m^{\*}(E\_{k })$ . (6)
 ii) Prove that if E1 and E2 are measurable sets then E1  E2 is measurable. (4)

4. i) If A is a measurable set of finite outer measure that is contained in B then Prove that
 m\*(B-A) = m\*(B) - m\*(A). (2)
 ii) Construct Cantor's ternary set in [1,0]. (2)
 iii) Show that Cantor's ternary set is uncountable. (3)
 iv) Prove that the Cantor's ternary set has outer measure zero. (3)

5. Let *f* and *g* be measurable functions defined on the same domain *E*.
 i) Prove that *f + g*  is measurable. (5)

 ii) Let {*fn*} be a sequence of measurable functions on *E* that converges point wise almost
 everywhere on *E* to the function *f* . Prove that *f* is measurable. (5)

 6. Prove linearity and monotonicity of Lebesgue integral for bounded measurable functions. (8+2)

 7. i) State and prove Bounded convergence theorem. (5)
 ii) Prove Lebesgue dominated convergence theorem. (5)

 8. i) Define a) Total variation of a real valued function.
 and b) Bounded variation of a function. (2+1)
 ii) Prove that a function f is of bounded variation on the closed interval [a , b] if and only if
 it is the difference of two increasing functions on [a , b]. (7)

9. i) Define absolute continuity of a function (2)
 ii) If f is absolutely continuous function on [a , b] and f '(x) = 0 almost everywhere
 on [a , b] then prove that f is constant. (8)

10. State and prove Riesz-Fisher theorem. (10)