Register Number:
Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
B.Sc. MATHEMATICS - IV SEMESTER
SEMESTER EXAMINATION: APRIL 2018
MT-415 MATHEMATICS IV

Time- 1 ½ hrs. **Max Marks-35**

This paper has one printed page.

Answer any seven questions. **(7x5=35)**

1. Prove that, the normalizer of any element of a group is a subgroup of the group. Also prove that centre of a group is a normal subgroup of a normalizer of any element.
2. Show that every factor group of a cyclic group is cyclic.
3. Let $f : G \to G'$ be a homomorphism from G into G' with Kernel K. Then prove that f is one-one if and only if $K = \{e\}$ where e is the identity element of G.
4. State and prove the fundamental theorem of homomorphism.
5. Find the Fourier series of $f(x) = e^x, -\pi < x < \pi.$
6. Find the Fourier series of $f(x) = x^2, -\pi \le x \le \pi.$ Deduce that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$
7. Find the Fourier half range sine series for the function $f(x) = (x-1)^2, 0 < x < 1.$
8. Expand $\log(1 + x + y)$ by Taylor's theorem at $(0,0).$
9. Test for maximum and minimum for the function $f(x, y) = x^3 y^2 (12 - x - y).$
10. Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$
