Register Number:
Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
B.Sc. MATHEMATICS - IV SEMESTER
SEMESTER EXAMINATION: APRIL 2018
MT-415 MATHEMATICS IV

Time- 1 ½ hrs. **Max Marks-35**

This paper has one printed page.

Answer any seven questions. **(7x5=35)**

1. Prove that a subgroup H of a group G is normal in G if and only if the product of two right cosets in G is again a right coset in G.
2. If f is a homomorphism from a group G into G' then prove that the range $f(G)=\{f(g) \mid g \in G\}$ is a subgroup of G'.
3. If $f:(C^*,\times) \to (C^*,\times)$ Where C^* is the set of non-zero complex numbers, defined by $f(a+ib)=a-ib$ then prove that f is an isomorphism and find its kernel.
4. State and prove the fundamental theorem of homomorphism.
5. Find the fourier series for the function $f(x)=\begin{cases} 0, & -2<x<0 \\ 1, & 0<x<2 \end{cases}$
6. Express $f(x)=\dfrac{(\pi-x)}{2}$ as a fourier series with period 2π to be valid in the interval 0 to 2π and deduce that $\dfrac{\pi}{4}=1-\dfrac{1}{3}+\dfrac{1}{5}-\dfrac{1}{7}+\cdots$.
7. Find the fourier half range cosine series for the function $f(x)=\begin{cases} x, & 0<x<\pi/2 \\ \pi-x, & \pi/2<x<\pi \end{cases}$
8. Expand $e^x \cos y$ by Taylor's Theorem near the point $\left(1,\pi/4\right)$ up to the second degree terms.
9. Examine the maximum and minimum values of the function $\sin x+\sin y+\sin(x+y)$
10. If the temperature T at any point $p(x,y,z)$ is $T=Axyz^2$ where A is a constant show that the highest temperature at a point on the sphere $x^2+y^2+z^2=1$ is $A/8$.
