Register Number:

Date: 17-11-2020

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
M.Sc. MATHEMATICS-III SEMESTER
SEMESTER EXAMINATION: NOVEMBER-2020
MT-9118: FUNCTIONAL ANALYSIS

Duration: 2.5 Hours

Max. Marks: 70

The paper contains ONE page and ONE part

I. ANSWER ANY SEVEN FULL OF THE FOLLOWING. **(7x10=70)**

1. State and prove Holder's and Minkowski's inequality for l_p^n−space [5+5]
2. Prove that l_∞ is a Banach space with the norm, $||x|| = \sup_i |x_i|, x \in l_\infty$, where $l_\infty = \{x = (x_1, x_2, \ldots, x_n, \ldots) | \sup |x_i| < \infty\}$ [10]
3. Let $C[0,1]$ be the space of all real valued continuous functions defined on $[0,1]$. Prove or disprove $C[0,1]$ is a Banach space with the norm $||f|| = \max\{|f(x)| : x \in [0,1]\}$ [10]
4. Show that a normed linear space is a Banach space if and only if $S = \{x : ||x|| = 1\}$ is complete. [10]
5. State and prove Riesz Lemma. [10]
6. Let X and Y be two normed linear spaces. Prove that the vector space $B(X,Y)$ the set of continuous linear transformations of X into Y is a normed linear space with respect to the pointwise linear operations and the norm defined by $||T|| = sup\{||T(x)|| : ||x|| \leq 1\}$. Further prove, if Y is a Banach space the show that $B(X,Y)$ is also a Banach space. [10]
7. State and prove Bessel's Inequality. [10]
8. (a) Will $C[a,b]$ the set of all continuous functions in $[a,b]$ with the norm $||f|| = \max\{|f(x)| : x \in [a,b]\}$ be an inner product space?

   (b) Will l^p space with norm $||x|| = (\sum |x_i|^p)^{1/p}$ be an inner product space for all p. If not for all p, the for what value of p will it be an inner product space? [5+5]
9. State and prove the uniqueness of Hahn-Banach extension theorem. [10]
10. State and prove Closed Graph theorem. [10]

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