Register Number :

Date : 21-11-2020

St. Joseph's College (Autonomous), Bangalore-27
M.Sc. Mathematics - III Semester
End Semester Examination: November- 2020
MTDE9418 – Mathematical Methods

Duration: 2 ½ hrs **Max. Marks: 70**

1. **The paper contains two pages.**
2. **Answer any SEVEN FULL questions.**

1. Find the solution of the Fredholm integral equation of second kind

$$u(x) = \cos x + \lambda \int_0^{\pi} \sin(x-t)\, u(t)\, dt.$$ [10M]

2. a) Find the solution of an integral equation with the aid of the resolvent kernel.

$$\phi(x) = e^{x^2} + \int_0^x e^{x^2-t^2} \phi(t)\, dt.$$ [5M]

b) Using the method of successive approximation solve the integral equation

$g(x) = 1 + \int_0^x g(t)\, dt$ by taking $g_0(x) = 0$. [5M]

3. a) Solve an integral equation $y(x) = \frac{x^2}{2} - \int_0^x y(t)(x-t)\, dt$ by Laplace transform method. [5M]

b) Transform the boundary value problem $\frac{d^2y}{dx^2} + xy = 1,\ y(0) = y(1) = 0$ into an integral equation. [5M]

4. a) Examine the asymptotic behaviour of $\int_x^{\infty} e^{-t^4} dt$ as $x \to +\infty$. [5M]

b) Find the leading term of the asymptotic expansion for the given

$$I(x) = \int_0^{\infty} e^{-x\sinh^2 t} dt \ as\ x \to \infty.$$ [5M]

5. State and prove Watson's lemma and Evaluate $\int_0^{10} \frac{e^{-xt}}{1+t} dt$ as $x \to \infty$ using Watson's lemma.

[10M]

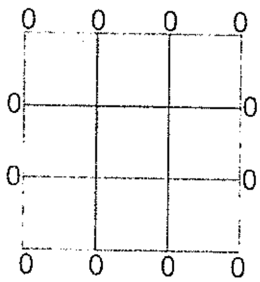
6. Solve the equation $\frac{dy}{dx} = 3x + \frac{y}{2}$ with $y(0) = 1$ at the points $x = 0.1$ & $x = 0.2$ taking step length $h = 0.1$ using Runge-kutta method of order four. **[10M]**

7. Use Taylor's series method to find y at $x = 0.1, 0.2, 0.3$. Apply Adams-Bashforth method for $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$ to calculate $y(0.4)$. **[10M]**

8. Using Crank-Nicholson method solve $u_t = u_{xx}$ subject to $u(x,0) = 100x(1-x)$, $u(0,t) = u(1,t) = 0$, $t > 0$, $0 < x < 1$. Compute u for one time step by taking $h = \frac{1}{4}$, $k = \frac{1}{64}$. **[10M]**

9. Solve the partial differential equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square with sides $x = y = 0, x = y = 3$ with $u = 0$ on the boundary and mesh length equal to 1.

[10M]



10. Evaluate the pivotal values of the equation $u_{tt} = 16u_{xx}$ taking $h = 1$, $upto$ $t = 1$. The boundary conditions are $u(0,t) = u(5,t) = 0$, $u_t(x,0) = 0$ and $u(x,0) = x^2(5-x)$.

[10M]

MTDE9418-A-20