Register Number:

Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
B.Sc. MATHEMATICS - II SEMESTER
EXAMINATION - APRIL 2018
MT 215: MATHEMATICS – II

Time:2 1/2 hrs **Maximum marks: 70**

This question paper has 2printed pages and 5 parts

I. ANSWER ANY FIVE QUESTIONS. **(5X2=10)**

1. In the group of set of positive rationals Q^+, $*$ is defined by a $*$ b = $\frac{ab}{5}$, find inverse of 4.
2. Define a semi-group with an example.
3. Show that for the curve $r = ae^{\theta \cot \alpha}$, where α is a constant, the tangent is inclined at a constant angle to the radius vector.
4. What is the length of the perpendicular from the pole to the tangent at P(r, θ) to the curve r = $f(\theta)$
5. Find the asymptotes of the curve $r\theta = a$
6. Write the formula to find the radius of curvature when the curve is

   i) y = f(x)

   ii) r = $f(\theta)$
7. Solve $(x^2 - ay)dx + (y^2 - ax)dy = 0$
8. Reduce the given equation $(x^2 - 1)p^2 - 2xyp + y^2 - 1 = 0$ to the form $y = px + f(p)$ and hence find its general solution.

II. ANSWER ANY THREE QUESTIONS **(3x 6 = 18)**

9. Prove that $G = \{3^n : n \text{ is an integer}\}$ is an abelian group under multiplication.

10. In a group G,Prove that
    a) The identity element of a group G is unique
    b) The inverse of an element in G is unique

11. Prove that G = { 1, 3, 4, 5, 9}is an abelian group under multiplication modulo 11.

12. Prove that a non empty subset H of a group (G, *) is a subgroup of G iff $\forall$ a,b$\in$ H, $a * b^{-1} \in$ H.

III. ANSWER ANY THREE QUESTIONS **(3x 6 = 18)**

13. Show that the curves $r = \frac{a}{1+\cos\theta}$ & $r = \frac{b}{1-\cos\theta}$ intersect orthogonally

14. Find the pedal equation of the curve $r^m = a^m \cos m\theta$.
15. Show that evolute of the parabola $y^2 = 4ax$ is $4(x - 2a)^3 = 27ay^2$
16. Find all the asymptotes of the curve $y^3 + x^2y + 2xy^2 - y + 1 = 0$
17. Discuss the position and nature of the double points on the curve $x^3 + x^2 + y^2 - x - 4y + 3 = 0$.

IV. ANSWER ANY ONE QUESTION **(1 x 6 = 6)**

18. Find the area bounded by the cissoid $y^2(a - x) = x^3$ and its asymptote

19. Find the Volume generated by revolving the curve Astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ about the x-axis

V. ANSWER ANY THREE QUESTIONS **(3x 6 = 18)**

20. Solve $\sin x \frac{dy}{dx} + y\cos x = x\sin x$
21. Solve $x\frac{dy}{dx} + y = y^2 \log x$
22. Solve $(x^2 - 3xy + 2y^2)dx + x(3x - 2y)dy = 0$
23. Find the orthogonal trajectories of the family of curves $x^2 + 3y^2 = cy$.