Register Number:

Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27
M.Sc. PHYSICS - II SEMESTER
SEMESTER EXAMINATION: APRIL 2018
PH 8415 - QUANTUM MECHANICS I

Time-2 1/2 hrs. **Maximum Marks-70**

This question paper has 4 printed pages and 2 parts

PART A

Answer any FIVE full questions. **MAX. MARKS 5x10=50**

(For some of the questions in Part A, you may need the values of constants listed in Part B).

1.
   (a) With a circuit diagram, describe the set up for photoelectric effect. (**2 Marks**)
   (b) Explain what photoelectric effect is. (**2 Marks**)
   (c) What is photo-ionization? Is it the same as photoelectric effect? (**3 Marks**)
   (d) Can we explain photoelectric effect classically? If yes, then how? If not, why? (**3 Marks**)

2.
   (a) High energy photons (γ rays) are incident on electrons that are at rest. If the photons get backscattered and their energies are much higher than the rest mass energy of the electrons, find the wavelength shift of the photons. (**5 Marks**)
   (b) A system is described by a wavefunction: $\psi_n(x, t=0) = A\,e^{-nx/a}$ $\quad \text{for } 0 < x < \infty$
   where n is the quantum number and whose associated eigenvalues are given as: $E_n = \epsilon_n$
      i. Normalize the wavefunction and compute A (**2 Marks**)
      ii. If the particle is in the m^{th} state at time $t=0$, what will be the wavefunction at a later time $t=\tau$? (**3 Marks**)

3.

(a) For a Linear Operator $\hat{A}$, show that the time evolution of its expectation value is given by: $\frac{d}{dt}\langle\hat{A}\rangle=\frac{1}{i\hbar}\langle[\hat{A},\hat{H}]\rangle+\left\langle\frac{\partial\hat{A}}{\partial t}\right\rangle$. (**3 Marks**)

(b) From the equation above, show that: $\frac{d}{dt}\langle\hat{X}\rangle=\frac{1}{m}\langle\hat{P}\rangle$ and $\frac{d}{dt}\langle\hat{P}\rangle=-\left\langle\frac{d\hat{V}}{dx}\right\rangle$ (**3 Marks**)

(c) What is the physical implication of the results in (b)? (**4 Marks**)

4. For a particle of mass m in a box of length L, the stationary states and eigenvalues are given as: $\psi_n(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right)$ and $E_n=\frac{\hbar^2}{2m}\frac{n^2\pi^2}{L^2}$. Before performing an experiment, the wavefunction $\Psi(x)$ is a superposition of all eigenstates.

(a) Write down the expression for $\Psi(x)$ (Assume that $\Psi(x)$ is normalized) (**2 Marks**)

(b) What will be the probability (according to your expression in (a) above) of finding the particle in state m? (**2 Marks**)

(c) Does your expression in (a) bring to mind any series representation? (**3 Marks**)

(d) What does the normalization condition mean for the coefficients in the expression for $\Psi(x)$? (**3 Marks**)

5. Consider a system described by a state $|\psi(t)\rangle=\frac{1}{6}\begin{pmatrix}1\\0\\4\end{pmatrix}$ and an observable

$\hat{A}=\begin{pmatrix}2&0&0\\0&1&i\\0&-i&1\end{pmatrix}$ What is the probability of getting 0 as a result of an experiment (i.e. finding the system in an eigenstate corresponding to eigenvalue 0)?

6. The radial component of the Schrodinger equation for Hydrogen atom works out to be:

$-\frac{\hbar^2}{2m}\frac{d^2u}{dr^2}+\left[-\frac{e^2}{4\pi\epsilon_0 r}+\frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right]u=Eu$ where u is the radial wavefunction.

(a) If we make the following variable changes: $\kappa=\sqrt{\frac{-2mE}{\hbar^2}}$ over $\rho_0=\frac{me^2}{2\pi\epsilon_0\hbar^2\kappa}$, what does the above Schrodinger equation reduce to? (**3 Marks**)

(b) What are the asymptotic points in the equation in (a)? (**1 Mark**)

(c) For each of the asymptotic regions, write down the reduced Schrodinger equation. (**3 Marks**)

(d) What are the forms of the wavefunction u in the asymptotic regions? (**3 Marks**)

7. In classical physics angular momentum of a particle is defined as: $\vec{L}=\vec{r}\times\vec{p}$.

(a) The above equation is in vector form. What are the cartesian components of $\vec{L}$ if $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ and $\vec{p}=p_x\hat{i}+p_y\hat{j}+p_z\hat{k}$. (**4 Marks**)

(b) Rewrite the above components in operator form using the following replacements: $x\rightarrow\hat{x}$, $y\rightarrow\hat{y}$, $z\rightarrow\hat{z}$, $p_x\rightarrow -i\hbar\frac{\partial}{\partial x}$, $p_y\rightarrow -i\hbar\frac{\partial}{\partial y}$ and $p_z\rightarrow -i\hbar\frac{\partial}{\partial z}$ (**6 Marks**)

PART B

Answer any FOUR full questions. **MAX. MARKS 4x5=20**

[Constants: h=6.6x10^{-34} J s (Planck's constant), 1eV = 1.6x10^{-19} J (electron volt to Joules), c=2.99x10^{8} m/s (speed of light),1Å = 1x10^{-10}m (Angstrom to meters), e = 1.6x10^{-19} C (electronic charge), m_{proton}=1.673x10^{-27}kg (mass of proton), $m_{electron}$=9.109x10^{-31}kg (mass of electron), a = 5.029x10^{-10}m (Bohr radius)]

8. A particle moving in one dimension is in a stationary state whose wave function is:

$$\psi(x)=\begin{cases}0 & x<0\\ A\left(1+\sin\frac{\pi x}{a}\right) & 0\le x\le a\\ 0 & x>a\end{cases}$$ where A and a are real constants.

Normalize $\psi(x)$ and find A in terms of a .

9. The Pauli spin matrices are: $\sigma_1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$, $\sigma_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$ and $\sigma_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$ .Show that $\sigma_i^2=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ for all matrices (these matrices represent the bases of angular momentum representation).

10. Consider a physical system described by the Hamiltonian $\hat{H}$. Let this Hamiltonian have eigenvalues ϵ_1 and ϵ_2 and eigenvectors (orthonormal to each other): $|A\rangle$ and $|B\rangle$. Before conducting an experiment, the system may be thought to be in a (superposed) state $|\Psi\rangle=m|A\rangle+n|B\rangle$

(a) What will be the expectation value: $\langle\hat{H}\rangle$ (**3 Marks**)

(b) What will be the probability that the experiment will yield a result implying that the system is in state $|B\rangle$? (**2 Marks**)

11. Let $\psi_n(x)$ be the orthonormal stationary states of a system with corresponding eigen energies E_n. Suppose that the normalized superposed wavefunction $\Psi(x)$ of the system is such that a measurement of the energy yields the value E_1 with probability $1/3$, E_2 with probability $3/8$ and E_3 with probability $7/24$. Write the expression for $\Psi(x)$ consistent with this information.

12. An operator $\hat{P}$ commutes with $\hat{J}_x$ and $\hat{J}_y$, the x and y components of an angular momentum operator. $\hat{J}_x$ and $\hat{J}_y$ satisfy the commutation relation: $[\hat{J}_x, \hat{J}_y] = i\hat{J}_z$ where $\hat{J}_z$ is the z component of the angular momentum. Show that $\hat{P}$ commutes with $\hat{J}_z$ (i.e. $[\hat{P}, \hat{J}_z] = 0$).

13. In a Compton scattering of photons from electrons at rest, if the photons scatter at an angle of 45° and if the wavelength of the scattered photons is $9\times10^{-13}\mathrm{m}$, find
    (a) the wavelength and the energy of the incident photons (**1 Mark**)
    (b) the energy of the recoiling electrons and the angle at which they recoil. (**4 Marks**)