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Register Number:

DATE: 11-04-2017

**ST. JOSEPH’S COLLEGE (AUTONOMOUS), BENGALURU-27**

**M.Sc. MATHEMATICS- IV SEMESTER**

**SEMESTER EXAMINATION- APRIL 2017**

**MT 0114: Measure and Integration**

**Time:** 2 ½ **Hours Max. Marks: 70**

**This question paper has two printed pages.**

**Answer Any SEVEN Questions**1**.** i) Define an outer measure. (1)
 ii) Prove that outer measure is monotone (2)
 iii) Prove that a countable set has outer measure zero. (3)
 iv) Prove that the outer measure is countably sub additive (4)

2. i) Prove that a countable set has outer measure zero. (3)
 ii) If $\left\{B\_{k}\right\}\_{k=1}^{\infty }$ is a descending collection of measurable sets and m(B1) $<\infty $
 then prove that $m\left(\bigcap\_{k=1}^{\infty }B\_{k}\right)=\lim\_{k\to \infty }m(B\_{k})$ . (7)

3. i) Prove that the interval (a , $\infty )$ is measurable for every a$ \in R$ . (7)
 ii) State Littlewood's three principles (3)

4. i) Define measurable function (2)
 ii) If f and g are two measurable functions defined on the same domain, prove that
 f + g , f + c , and fg are measurable where c is a constant (8)

5. i) State and prove Fatou's lemma (7)
 ii) Prove monotone convergence theorem (3)

6. i) Let $\left\{E\_{k}\right\}\_{k=1}^{n}$ be a finite disjoint collection of measurable subsets of the set of finite measure.
 For , let ak  be a real number. If  on E, then prove that
 . (4)

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 ii) Let  and be simple functions defined on a set of finite measure E. Then for any α and β,
 prove that . (4)
 iii) If    on E then prove that  (2)

7. State and prove Vitali's Covering lemma (10)

8. State and prove Lebesgue Dominated Convergence Theorem (10)

9. Prove that a function f is an indefinite integral if and only if it is absolutely continuous (10)

10. Prove that LP spaces (1  p <) are complete (10)