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# ST.JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. PHYSICS – II SEMESTER

SEMESTER EXAMINATION – APRIL 2017

## PH 8115 : ELECTRODYNAMICS

**Time: 2.5 hours** **Maximum Marks:70**

This question paper contains 2 parts and 3 printed pages

Some useful Identities:

$$\vec{\nabla}.(\vec{A} x \vec{B}) = \vec{B}.(\vec{\nabla} x \vec{A}) - \vec{A}.(\vec{\nabla} x \vec{B})$$

$$\vec{\nabla} x (\vec{A} x \vec{B}) = (\vec{B}.\vec{\nabla})\vec{A} - (\vec{A}.\vec{\nabla})\vec{B} + \vec{A}(\vec{\nabla}.\vec{B}) - \vec{B}(\vec{\nabla}.\vec{A})$$

**In Spherical polar co-ordinates**

$$\nabla t = \frac{\partial t}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial t}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial t}{\partial \varphi}\hat{\varphi}$$

$$\nabla x \mathbf{v} = \frac{1}{r\sin\theta}[\frac{\partial}{\partial\theta}(\sin\theta\, v_\varphi) - \frac{\partial v_\theta}{\partial\varphi}]\hat{r} + \frac{1}{r}[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\varphi} - \frac{\partial(r v_\varphi)}{\partial r}]\hat{\theta} + \frac{1}{r}[\frac{\partial(r v_\theta)}{\partial r} - \frac{\partial v_r}{\partial\theta}]\hat{\varphi}$$

*All bold capital letters denote vectors.*

## Part-A

**Answer any 5 questions. Each question carries 10 marks. (10x5=50)**

1. Suppose we have a piece of magnetised material with magnetisation **M** . Justify that the potential of this magnetised object is same as that produced by a volume current $\vec{J}_b = \nabla x \vec{M}$ throughout the material plus a surface current $\vec{K} = \vec{M} x \hat{n}$ on the boundary by deriving the above mentioned equation. Hence, derive ampere's law for a magnetised material. (6+4)

2. a) Write Maxwell's equations in differential form. Explain what each equation signifies.
b) Using these equations written above, derive these equations for material medium. (6+4)

3. a) Derive the wave equations for **E** and **B** for propagation of E.M. waves in conducting medium from Maxwell's equations in material medium assuming that the free charge is zero. (The fields **E** and **B** obey same form of equation, hence derivation of any one of them will suffice; write the other directly).
b) If the plane wave equations of **E** and **B** are solutions to their wave equations, show that the wave number ' $\tilde{k}$ ' is a complex quantity and is given as $\tilde{k} = k + i\kappa$ .Hence, find skin depth in the medium in terms of these variables. (No need to derive equations for values of k and κ.) (4+6)

4. a) Define Poynting vector and interpret it's direction from the expression.What does it signify?
b) Find the approximate potential due to an electric dipole at a point far off from the dipole.

(3+7)

5. Consider an oscillating dipole made up of two tiny charged metal spheres with charge +q(t) and –q(t) separated by a distance 'd' oscillating with angular frequency ω. The potentials at a point 'P' at time 't' in the far radiation zone ( d<<λ<<r) are given as

$$V(r,\theta,t)=\frac{-p_o\omega}{4\pi\epsilon_o c}\left(\frac{\cos\theta}{r}\right)\sin\left[\omega\left(t-\frac{r}{c}\right)\right] \qquad \vec{A}(r,\theta,t)=\frac{-\mu_o p_o\omega}{4\pi r}\sin\left[\omega\left(t-\frac{r}{c}\right)\right]\hat{z}$$

where 'r' is the distance from centre of dipole to point 'P' and θ is the acute angle between 'd' and 'r'. Find the fields '**E**' and '**B**' at point 'P'. ( Hint: Solve in spherical polar co-ordinates) and the intensity of radiation radiated by the dipole. (10)

6. If the scalar and vector potential due to sources ρ and **J** are given as

$$V(r,t)=\frac{1}{4\pi\epsilon_o}\int\frac{\rho(\boldsymbol{r}')}{R}d\tau' \qquad \boldsymbol{A}(\boldsymbol{r})=\frac{\mu_o}{4\pi}\frac{\int \boldsymbol{J}(\boldsymbol{r}')}{R}d\tau'$$

where **R** is the distance from source point **r'** to field point **r** .

a) Comment on how and why these equations change when the electromagnetic news from the source travels to the field point(non-static case).

b) If this changed scalar potential also obeys Lorentz gauge $\nabla^2 V-\mu_o\epsilon_o\frac{\partial^2 V}{\partial t^2}=\frac{-\rho}{\epsilon_o}$ then we can justify that our argument for changing these equations is correct.

Assuming that the same argument holds for the scalar and vector potentials, show that this changed scalar potential obeys Lorentz gauge. (4+6)

7. a) Show that work-energy theorem holds in relativistic dynamics.

b) Considering force **F** to be the derivative of momentum with respect to ordinary time, show how the various components of this force transform from reference frame $S$ to $\bar{S}$ . If the particle is instantaneously at rest in $S$ then how does the component of force i) parallel and ii) perpendicular to the motion of $\bar{S}$ transform ? Given : the transformation matrix M for transforming from $S$ to $\bar{S}$ is

$$M=\begin{pmatrix}\gamma & -\gamma\beta & 0 & 0\\ -\gamma\beta & \gamma & 0 & 0\\ 0 & 0 & 1 & 1\\ 0 & 0 & 0 & 1\end{pmatrix}$$

where $\gamma=\frac{1}{\sqrt{(1-v^2/c^2)}}$ $\beta=v/c$ and v is the velocity of $\bar{S}$ relative to $S$ (4+6)

## Part-B

**Answer any 4questions. Each question carries 5 marks. (4x5=20)**

8. The electrostatic potential V(x, y) in free space in a region where the charge density ρ is zero is given by $V(x,y)=4e^{2x}+f(x)-3y^2$ . Given that the x-component of the electric field $E_x$ and V are zero at the origin, find f(x).

9. A plane electromagnetic wave is propagating in a lossless dielectric. The electric field is given by $\vec{E}(x,y,z,t)=E_o(\hat{x}+A\hat{z})\exp[ik_o[-ct+(x+\sqrt{3}z)]]$
where c is the speed of light in vacuum, $E_0$ , A and $k_0$ are constant and $\hat{x}$ and $\hat{z}$ are unit vectors along the x- and z-axes. Find the relative dielectric constant of the medium $\varepsilon_r$ and the constant A.

10. Consider a long straight wire in which a time dependent slowly varying current $I = I_0\sin\omega t$ flows down. Find the magnitude and direction of the electric field **E**(s,t) at a perpendicular distance 's' from the wire of radius 'a' assuming that the field goes to 0 as $r\rightarrow\infty$ . Also, find the displacement current.

11. A non-relativistic particle of mass m and charge e , moving with a velocity $\vec{v}$ and acceleration $\vec{a}$ , emits radiation of intensity I . What is the intensity of the radiation (calculate in terms of I) emitted by a particle of mass m / 2 , charge e/2 , velocity $\vec{v}/2$ and acceleration $4\vec{a}$ ?

12. Assuming that the real and imaginary parts of ' $\tilde{k}$ ' are given as: $k=\omega\sqrt{\frac{\epsilon\mu}{2}}[\sqrt{[1+(\frac{\sigma}{\epsilon\omega})^2]}+1]^{(1/2)}$ and $\kappa=\omega\sqrt{\frac{\epsilon\mu}{2}}[\sqrt{[1+(\frac{\sigma}{\epsilon\omega})^2]}-1]^{(1/2)}$ , calculate the minimum thickness of silver coating required in designing a safe microwave experiment to operate at a frequency of 30 GHz. Given that the resistivity of silver is 1.59 x $10^{-8}$ Ω-m and refractive index of silver is 0.15. Given : $\mu_o=4\pi x 10^{-7} H/m$ and $\epsilon_o=8.85 x 10^{-12} F/m$ .

13. a) If the electric and magnetic fields are unchanged when the potential **A** changes (in suitable units) according to $\bar{A}=\vec{A}+\hat{r}$ , where $\vec{r}=r(t)\hat{r}$ , then what should the scalar potential V simultaneously change to?
b) Show that the four- dimensional scalar product is invariant under Lorentz transformation for transforming from reference frame $S$ to $\bar{S}$ . (2+3)