Register Number: DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. PHYSICS - II SEMESTER

SEMESTER EXAMINATION- APRIL 2017

PH 8415: QUANTUM MECHANICS I

Time: 2.5 hrs. **Maximum Marks: 70**

This question paper has 3 printed pages and 2 parts

PART A **MAX. MARKS 5x10=50**

Answer any FIVE full questions.

1. What is the maximum change in frequency due to Compton Effect. Assuming Compton Effect to take place with visible light, will we be able to see change in color as we view Compton effect at different angles (take any one frequency for colors given)? Your argument should be made in terms of frequencies (rough wavelengths values of various colors are violet: 380-450 nm; blue: 450-495 nm; green: 495-570 nm; orange: 570-590 nm; yellow: 590-620; red: 620-750 nm). (**10 Marks**)

2.
   (a) Are the following expressions valid? Provide arguments (derivations)
   i. $[\hat{A},\hat{B}\hat{C}]=[\hat{A},\hat{B}]\hat{C}+\hat{B}[\hat{A},\hat{C}]$ (**3 Marks**)
   ii. $[\hat{A}\hat{B},\hat{C}]=[\hat{A},\hat{C}]\hat{B}+\hat{A}[\hat{B},\hat{C}]$ (**3 Marks**)
   (b) Show that if $\hat{A}$, $\hat{A}$ and $\hat{A}$ are linear operators, then: $[\hat{A},[\hat{B},\hat{C}]]+[\hat{B},[\hat{C},\hat{A}]]+[\hat{C},[\hat{A},\hat{B}]]=0$ (**4 Marks**)

3. A particle confined in a box must have a certain minimum energy (non-zero) called the zero point energy. Comment on how this is consistent with quantum mechanical principle and different from classical expectations. (**10 Marks**)

4. The Schrodinger equation is given as: $i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t)=-\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r},t)+V\,\Psi(\vec{r},t)=E\,\Psi(\vec{r},t)$. For a free particle, $V=0 \quad -\infty<x<\infty$. The generalized solution to this problem is given as: $\psi_n(x)=A\,e^{\frac{i}{\hbar}px}$.
   (a) Explain how you will normalize this solution. (**2 Marks**)
   (b) Using this problem as example, state and explain the postulates (6 postulates with not more than two sentences each please; a mathematical expression will suffice in most cases) of quantum mechanics. (**8 Marks**)

5. Let K be the operator defined by $K=|\varphi\rangle\langle\psi|$, where $|\varphi\rangle$ and $|\psi\rangle$ are two vectors of the state space.
   (a) What is the condition for K to be Hermitian. (**5 Marks**)
   (b) Compute K^2 . What is condition for K to be a projection operator? (**5 Marks**)

6. The expression for the ladder operator for the Simple Harmonic Oscillator is given as: $\hat{a}=\sqrt{\frac{m\omega}{2\hbar}}\hat{x}+\boldsymbol{i}\frac{1}{\sqrt{m\omega\hbar}}\hat{p}$.
   (a) Compute the products:
      i. $\hat{a}\hat{a}^{\dagger}$ (**3 Marks**)
      ii. $\hat{a}^{\dagger}\hat{a}$ (**3 Marks**)
   (b) Express the Hamiltonian for the Simple Harmonic Oscillator using the products (one or both). (**4 Marks**)

7. The angular momentum in classical mechanics is: $\vec{L}=\vec{r}\;x\,\vec{p}$.
   (a) Express this in component form using $\vec{r}=(x,y,z)$ and $\vec{p}=(p_x,p_y,p_z)$. (**2 Marks**)
   (b) If (in quantum mechanics we borrow the expressions from classical mechanics and convert to operator form; note the hats to the variables) $[\hat{x},\hat{p}_x]=i\hbar$, $[\hat{y},\hat{p}_y]=i\hbar$ and $[\hat{z},\hat{p}_z]=i\hbar$ and noting that $\hat{p}=-i\hbar\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)$, obtain:
      i. $[\hat{x},\hat{L}_y]$ (**4 Marks**)
      ii. $[\hat{p}_y,\hat{L}_z]$ (**4 Marks**)

PART B **MAX. MARKS 4x5=20**

Answer any FOUR full questions.

[Constants: h=6.6×10^{-34} J s (Planck's constant), 1eV = 1.6×10^{-19} J (electron volt to Joules), c=2.99×10^{8} m/s (speed of light),1Å = 1×10^{-10}m (Angstrom to meters), e = 1.6×10^{-19} C (electronic charge), m_{proton}=1.673×10^{-27}kg (mass of proton), $m_{electron}$=9.109×10^{-31}kg (mass of electron), a = 5.029×10^{-10}m (Bohr radius)]

8. An electron of mass m and charge e initially at rest gets accelerated by a constant electric field E . Compute the ***rate of change*** of de-Broglie wavelength of this electron at time t , ignoring relativistic effects. (**5 Marks**)

9. The state of a free particle is represented by the wave function $\psi(x,0)=N e^{-x^2/2a^2+ik_0 x}$.
Compute the value of N. You may use the formula for a Gaussian integral:
$\int_{\infty}^{\infty} e^{-\alpha t^2} dt = \sqrt{\frac{\pi}{\alpha}}$. (**5 Marks**)

10. Consider a particle of mass m in a potential: $V(x)=\begin{cases}\infty, & if \quad x<0, \quad x>L \\ 0 & if \quad 0<x<L\end{cases}$.
$|u_n\rangle$ are eigenstates of the Hamiltonian H of the system, and their eigenvalues are $E_n=\frac{\hbar^2}{2m}\frac{\pi^2}{L^2}n^2$. The state of a particle at instant $t=0$ is $|\psi(0)\rangle=a_1|\varphi_1\rangle+a_2|\varphi_2\rangle+a_3|\varphi_3\rangle+a_4|\varphi_4\rangle$

(a) What is the probability of finding the particle in a level having energy less than $\frac{\hbar^2}{2m}\frac{\pi^2}{L^2}3$ (**2 Marks**)

(b) When the energy is measured, a value $\frac{\hbar^2}{2m}\frac{\pi^2}{L^2}8$ is obtained. After the measurement, what is the state of the system? What is the result if the energy is measured again (immediately)? (**3 Marks**)

11. If the electron were a classical solid sphere with radius: $r_c=\frac{e^2}{4\pi\epsilon_0 m c^2}$ and its angular momentum $1/2\hbar$, how fast would the electron be rotating? (obtain both the linear and angular velocity). Does this make sense? (**5 Marks**)

12. The ground state wavefunction of a Simple Harmonic Oscillator is $\psi_0(x)=\left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{\frac{-m\omega}{2\hbar}x^2}$.
Using the ladder operator obtain the first excited state wavefunction. (**5 Marks**)

13. The wavefunction of a particle confined in a box of length L is $\psi(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{\pi x}{L}\right)$ where $0<x<L$.

(a) Sketch this wavefunction (**2 Marks**)

(b) Calculate the probability of finding the particle in the region $0<x<L/2$ (**3 Marks**)