Register Number:
DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BANGALORE-27

M.Sc. PHYSICS - II SEMESTER

SEMESTER EXAMINATION- APRIL 2017

PH 8415: QUANTUM MECHANICS I

Time: 2.5 hrs. **Maximum Marks: 70**

This question paper has 4 printed pages and 2 parts

PART A **MAX. MARKS 5x10=50**

Answer any FIVE full questions.

1. A photon of violet light (of wavelength 420 nm) collides with an electron and is back scattered ($\theta=180^\circ$).
   (a) Compute the energy transferred to the electron (in eV) (**3 Marks**)
   (b) Had the violet light interacted with an electron in sodium metal (having work-function of 2.28 eV) in photoelectric effect, what would have been the energy of the electron? (**4 Marks**)
   (c) Explain whether the violet light photon can "knock off" an electron from a metal through Compton Effect. (**3 Marks**)

2.
   (a) Show that the linear operator $\left(\frac{d}{dx}+x\right)\left(\frac{d}{dx}-x\right)$ is equal to $\frac{d^2}{dx^2}-x^2-1$ (**5 Marks**)
   (b) Show that xe^{-x^2} is an eigenfunction of the linear operator: $\frac{d^2}{dx^2}-4x^2$. What is the eigenvalue? (**5 Marks**)

3. With a diagram of the setup and figures for the results (characteristics), explain Photoelectric Effect. Argue as to whether you consider this phenomenon to prove the particle nature of light or it's wave nature.

(**10 Marks**)

4.

(a) The Schrodinger equation is given as:

$i\hbar\frac{\partial}{\partial t}\Psi(\vec{r},t)=-\frac{\hbar^2}{2m}\nabla^2\Psi(\vec{r},t)+V\,\Psi(\vec{r},t)=E\,\Psi(\vec{r},t)$. For a particle in a one dimensional box, the potential is given by the expression: $V(x)=\begin{cases}\infty, & if \quad x<0, \quad x>L\\ 0 & if \quad 0<x<L\end{cases}$. The ortho-normalized state for this problem is given as: $\psi_n(x)=\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$ with energy eigenvalues: $E_n=\frac{\hbar^2}{2m}\left(\frac{\pi^2}{L^2}\right)n^2$. Using this problem as example, state and explain the postulates (6 postulates with not more than two sentences each please; a mathematical expression will suffice in most cases) of quantum mechanics.

(**8 Marks**)

(b) In relation to your statements.:

i. what is the probability of finding the particle in the $n=3$ state. (**1 Mark**)

ii. Assuming the particle to be in the state $n=3$ at time $t=0$, what is the wave function after 2 seconds of evolving the system in time? (**1 Mark**)

5. The state space of a certain physical system is spanned by the orthonormal basis: $|u_1\rangle$, $|u_2\rangle$, $|u_3\rangle$. The kets $|\psi_0\rangle$ and $|\psi_1\rangle$ are given as: $|\psi_0\rangle=\frac{3}{5}|u_1\rangle-i\frac{1}{2}|u_2\rangle+\frac{\sqrt{39}}{10}|u_3\rangle$ and $|\psi_1\rangle=\frac{1}{\sqrt{3}}|u_1\rangle+\frac{\sqrt{5}}{3}|u_2\rangle+\frac{i}{\sqrt{3}}|u_3\rangle$

(a) Are $|\psi_0\rangle$ and $|\psi_1\rangle$ normalized? (**2 Marks**)

(b) Compute:

i. $|\psi_0\rangle\langle\psi_0|$ (**4 Marks**)

ii. $|\psi_0\rangle\langle\psi_1|$ (**4 Marks**)

6. The Hamiltonian for a Simple Harmonic Oscillator is given as: $\hat{H}=\frac{\hat{p}^2}{2m}+\frac{1}{2}m\omega^2\hat{x}^2$ where the terms have their usual meaning and the carets atop variables imply operators.

(a) Express the Hamiltonian in terms of a product of two operators (ladder operators): $\hat{a}$ and $\hat{a}^\dagger$. (**4 Marks**)

(b) Compute the product of these two ladder operators:

i. $\hat{a}\hat{a}^\dagger$ (**3 Marks**)

ii. $\hat{a}^\dagger\hat{a}$ (**3 Marks**)

7. The angular momentum in classical mechanics is: $\vec{L}=\vec{r}\, x\, \vec{p}$.
   (a) Using $\vec{r}=(x,y,z)$ and $\vec{p}=(p_x,p_y,p_z)$, show that $L^2=(r\,x\,p)^2=r^2p^2-(\vec{r}\bullet\vec{p})^2$ **(2 Marks)**
   (b) If (in quantum mechanics we borrow the expressions from classical mechanics and convert to operator form; note the hats to the variables), in spherical polar coordinates the angular momentum components are represented as: $\hat{L}_x=i\hbar\left(\sin\varphi\frac{\partial}{\partial\theta}+\cot\theta\cos\varphi\frac{\partial}{\partial\varphi}\right)$, $\hat{L}_y=i\hbar\left(-\cos\varphi\frac{\partial}{\partial\theta}+\cot\theta\sin\varphi\frac{\partial}{\partial\varphi}\right)$ and $\hat{L}_z=-i\hbar\frac{\partial}{\partial\varphi}$ and that $\hat{L}^2=\hat{L}_x{}^2+\hat{L}_y{}^2+\hat{L}_z{}^2$. Obtain the expression for $\hat{L}^2$ in spherical polar coordinates. **(8 Marks)**

PART B **MAX. MARKS 4x5=20**

Answer any FOUR full questions.

[Constants: **h**=6.6x10^{-34} J s (Planck's constant), **1eV** = 1.6x10^{-19} J (electron volt to Joules), **c**=2.99x10^{8} m/s (speed of light),**1Å** = 1x10^{-10}m (Angstrom to meters), **e** = 1.6x10^{-19} C (electronic charge), $\mathbf{m_{proton}}$=1.673x10^{-27}kg (mass of proton), $\mathbf{m_{electron}}$=9.109x10^{-31}kg (mass of electron), **a** = 5.029x10^{-10}m (Bohr radius), **Avogadro Number**=6.022x10^{23}/mol]

8. The work-function of Platinum is 612.66 kJ/mol.
   (a) What is the minimum energy required to “knock off” an electron from this metal. **(1 Mark)**
   (b) What is the maximum wavelength of light that can eject an electron with this energy? **(1 Mark)**
   (c) If we shine green light (wavelength of 5500 Å) on this metal, will electrons be ejected? **(1 Mark)**
   (d) What should be the value of the work-function for green light of the above wavelength to eject electrons? **(2 Marks)**

9. The wave function in the ground state of hydrogen atom is given as: $\psi=Ae^{-r/a}$ where r measures distance from nucleus and a is the Bohr radius. Compute A. You may use the integral $\int_0^\infty e^{-bt}\,dt=\frac{1}{b}$. **(5 Marks)**

10. A physical system whose three dimensional state space is formed by the orthonormal basis by the three kets: $|u_1\rangle$, $|u_2\rangle$ and $|u_3\rangle$ is described by the Hamiltonian $\boldsymbol{H}=\hbar\omega_0\begin{pmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 2\end{pmatrix}$ where ω_0 is a positive constant. The system is at time $t=0$ in the state: $|\psi(0)\rangle=\frac{1}{\sqrt{2}}|u_1\rangle+\frac{1}{2}|u_2\rangle+\frac{1}{2}|u_3\rangle$. At time $t=0$ the energy of the system is measured.

(a) What values can be found? (**1 Mark**)

(b) What are the probabilities of finding those values? (**2 Marks**)

(c) What is the value of $\langle H\rangle$? (**2 Marks**)

11. For Hydrogen atom, the potential energy is given as $V(r)=\frac{-e^2}{4\pi\epsilon_0}\frac{1}{r}$. Substituting this into the Schrodinger equation and separation of variables etc. yields the energy value from the radial equation to be: $E_n=-\left[\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\epsilon_0}\right)\right]\frac{1}{n^2}$ and the Bohr radius to be $a=\frac{4\pi\epsilon_0\hbar^2}{me^2}$. Using the Hydrogen atom as an analogy, consider the earth-sun system:

(a) What is the potential energy function (let m be the mass of earth and M the mass of the sun). (**1 Mark**)

(b) What is the "Bohr radius" a_g for this system? With $G=6.674\times10^{-11} m^3 kg^{-1} s^{-2}$ (Gravitational constant), $M_\odot=1.9891\times10^{30}$ kg (Solar mass), $R_\odot=6.9\times10^{8}$ m, σ = 5.67×10^{-8} W m^{-2} K^{-4} (Stefan-Boltzmann constant), $M_{Earth}=5.97\times10^{27}$kg (Mass of Earth), $D_{earth\text{-}sun}=1.49\times10^{11}$m (Earth-Sun distance)] work out the actual number of a_g (**4 Marks**)

12. Given that $\hat{a}\hat{a}^\dagger\psi_n=n\psi_n$ and $\hat{a}^\dagger\hat{a}\psi_n=(n+1)\psi_n$, find the expectation value of the potential energy in the nth state of the harmonic oscillator. (**5 Marks**)

13. The wavefunction of a particle confined in a box of length L is $\psi(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{3\pi x}{L}\right)$ where $0<x<L$.

(a) Sketch this wavefunction (**2 Marks**)

(b) Calculate the average value of position: $\langle x\rangle$ (**3 Marks**)