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Register Number:

DATE: **19-4-2017**

**ST. JOSEPH’S COLLEGE (AUTONOMOUS), BANGALORE-27**

**B.Sc. MATHEMATICS – VI SEMESTER**

**SEMESTER EXAMINATION: APRIL 2017**

MT 6112 : Mathematics - VII

Time- 3 hrs Max Marks-100

 **This question paper has three printed pages.**

**I. ANSWER ANY EIGHT OF THE FOLLOWING (8X2=16)**

1) Evaluate where 

2) Evaluate 

3) Find area of circle  using double integral.

4) Evaluate 

5) Evaluate 

6) State Gauss Divergence theorem.

7) Let and Find the derived set of A.

8) Let Is a neighbourhood of 3? Justify your

 answer.

9) Define Vector space.

10) Find , so that the vectors andare linearly independent in $V\_{3}\left(R\right).$

11) If V($R)$ is the vector space of all real valued functions and  is a

 subset of , then find the dimension of 

12) If V($R)$ is the vector space of all real square matrices of order and is a fixed element of

 , then show that the mapping defined by is a linear

 transformation, for any in.

**MT-6112-A-17**

**II. ANSWER ANY SEVEN OF THE FOLLOWING (7X6=42)**

13) Evaluate where is the arc of the helix which joins

 the points  and 

14) If is any curve leading from  and  show that

 

15) Evaluate  where D is the region bounded by the parabolas and

 .

16) Evaluate by interchanging the order of integration 

17) Evaluate  over the positive octant of sphere  by

 transforming into cylindrical polar co-ordinates.

18) Evaluate  where  and is the portion of the plane

  in first octant.

19) Evaluate where  and  is the volume enclosed by planes

  and surface 

20) State and prove Green’s theorem.

21) Evaluate using Gauss Divergence theorem  taken over

 the region bounded by 

22) Evaluate using Stoke’s theorem ****where  is the boundary of

 the rectangle 

**III. ANSWER ANY TWO OF THE FOLLOWING (2X6=12)**

23) Let be a topological space. Prove that a set  is open if and only if it is a

 neighbourhood of each of its points.

24) Prove that  where  is a subset of a topological space, is the derived

 set of and  is the closure of 

25) If****is a collection of subsets of , Find the basis 

 induced by . Also find the topology induced by .

**IV. ANSWER ANY FIVE OF THE FOLLOWING (5X6=30)**

26) Prove that a nonempty subset  of a vector space  is a subspace of  if and only if

  and  Verify whether is a

 subspace of $R^{3}$ over $R.$

27) If is a nonempty subset of a vector space, then prove that is a subspace of and it

 is the smallest subspace containing .

28) Prove that every finite set of nonzero vectors of a vector space has a linearly

 independent subset which spans the same subspace.

29) Define basis and dimension of a vector space. Verify whether the subset

 is linearly independent in the real vector

 space$V\_{4}(R)$. Also find a basis and dimension of 

30) Determine a linear transformation$ T:V\_{2}(R)\rightarrow V\_{3}(R)$ such that is the matrix of

 relative to the bases and of$ V\_{2}(R)$and

$ V\_{3}\left(R\right)$respectively.

31) State and prove Rank-Nullity Theorem.

32) Find a linear transformation $T:R^{3}\rightarrow R^{3} $whose range space is spanned by the vector

 and.