Register Number:
DATE:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
M.Sc. MATHEMATICS – III SEMESTER
SEMESTER EXAMINATION: OCTOBER 2021
(Examination conducted in JANUARY-MARCH 2022)

MT9218: CLASSICAL AND CONTINUUM MECHANICS

Time- 2 ½ hrs. **Max Marks-70**

The paper contains TWO pages.

Answer any SEVEN full questions. Each carrying 10 marks.

1. a) Find velocity and acceleration of the particle given by $r = 2e^{\omega t} sin\omega t\ ; \theta = \omega t,$ $where\ \omega\ is\ a\ constant.$

   b) Derive the expression for velocity in cylindrical co-ordinate system. (7+3)

2. a) Derive the expression for Coriolis force.

   b) The position vector of two point masses 100kg and 50kg are (3,-2,-4) and (-3,6,-5) respectively. Find the position of the center of mass. (8+2)

3. a) For a system of particles derive the expression for conservation of energy.

   b) A 2000kg empty rail cart moves east at 15m/s. A 50kg rock is dropped straight down into the moving cart. What is the final speed of the cart? (8+2)

4. a) State and prove Hamilton's principal for holonomic constraints.

   b) Solve the Poison's bracket of $\{|r|, |p|\} = \left\{(x^2 + y^2 + z^2)^{\frac{1}{2}}, ({P_x}^2 + {P_y}^2 + {P_z}^2)^{\frac{1}{2}}\right\}$. (7+3)

5. a) If $D = det(a_{ij})$. Verify that $\varepsilon_{ijk}\varepsilon_{pqr}D = \begin{vmatrix} a_{ip} & a_{iq} & a_{ir} \\ a_{jp} & a_{jq} & a_{jr} \\ a_{kp} & a_{kq} & a_{kr} \end{vmatrix}$

   Hence deduce the following results:

   i) $\varepsilon_{ijk}\varepsilon_{pqr} = \begin{vmatrix} \delta_{ip} & \delta_{iq} & \delta_{ir} \\ \delta_{jp} & \delta_{jq} & \delta_{jr} \\ \delta_{kp} & \delta_{kq} & \delta_{kr} \end{vmatrix}$

   ii) $\varepsilon_{ijk}\varepsilon_{pqk} = \delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp}$

   iii) $\varepsilon_{ijk}\varepsilon_{pjk} = 2\delta_{ip}$

   iv) $\varepsilon_{ijk}\varepsilon_{ijk} = 6$

b) Prove the vector identity using suffix notation

$(a \times b) \cdot (c \times d) = (a \cdot c)(b \cdot d) - (a \cdot d)(b \cdot c)$ (7+3)

6. a) Given a x_i- system, a vector $'a'$ has components $a_1 = -1, a_2 = 0, a_3 = 1$ and a tensor $\vec{A}$ has its matrix $[a_{ij}] = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 2 \\ 0 & -2 & 0 \end{bmatrix}$. The x'_i- system is obtained by rotating the x_i-system about the x_3- axis through an angle of 45^0 in the sense of a righthanded screw. Find the components of $'a'$ and $\vec{A}$ in x'_i- system.

b) State and prove Gauss Divergence theorem for a tensor. (5+5)

7. a) Find the velocity and acceleration field in both material and spatial form for the system of equation $x_1^0 = x_1 \cos \alpha t - x_2 \sin \alpha t$ and $x_2^0 = x_1 \sin \alpha t + x_2 \cos \alpha t$.

b) For the deformation defined by the system of equations

$x_1 = \alpha\, x_1^0 + \beta\, x_2^0,\ x_2 = -\alpha\, x_1^0 + \beta\, x_2^0,\ x_3 = \gamma\, x_3^0$ .Find $F, J\ and\ F^{-1}$. (5+5)

8. a) Derive the expression for normal strain in spatial description.

b) Find the path and stream lines for the motion define by velocity components

$v_1 = \frac{x_1}{1+t}\ , v_2 = \frac{2x_2}{1+t}\ \ and\ v_3 = \frac{3x_3}{1+t}$. (4+6)

9. a) Derive the expression for Reynold's transport formula.

b) Show that the motion of a continuum in circulation is preserved if and only if the acceleration is an irrotational vector. (6+4)

10. a) Find the value of k such that $v_1 = kx_3(x_2 - 2)^2\ , v_2 = -x_1x_2$ and $v_3 = kx_1x_3$, where the velocity components of an incompressible continuum is $div\ \vec{v} = 0$.

b) For a continuum body derive the expression for conservation of linear momentum.

(4+6)