Date:
Registration number:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
M.Sc MATHEMATICS - III SEMESTER
SEMESTER EXAMINATION: OCTOBER 2021
(Examination conducted in JANUARY-MARCH 2022)

MTDE9418 – MATHEMATICAL METHODS

Time- 2 ½ hrs Max Marks- 70

This question paper contains **TWO** printed pages.

Answer any **SEVEN FULL** questions. **7 x10=70 Marks**

1. a) Solve the Fredholm integral equation of second kind by the method of separable kernels, given that $u(x) = e^x + \lambda \int_0^1 2e^x e^t u(t)\,dt.$ [5M]

   b) Find the iterated kernel $K_1(x,t),\ K_2(x,t), K_3(x,t)$ for the Volterra integral equation with kernel $K(x,t) = \frac{2+\cos x}{2+\cos t}.$ [5M]

2. a) Find the resolvent kernel for the integral equation $\phi(x) = x^2 + \int_0^x e^{t-x}\phi(t)\,dt.$ [5M]

   b) Solve the integral equation $u(x) = 1 + 2\sin x - \int_0^x u(t)\,dt$ using Laplace Transform method. [5M]

3. Find eigen values and the corresponding eigen functions of an integral equation $y(x) = \lambda \int_0^{2\pi} \sin(x+t)\, y(t)\,dt$ with degenerate kernel. [10M]

4. a) Derive the small x behaviour of $\int_0^1 \frac{\sin(tx)}{t}dt\ \ as\ x \to 0.$ [5M]

   b) Given $I(x) = \int_0^\infty e^{-x\sinh^2 t}dt\ \ as\ \ x \to \infty,$ find the leading term of the asymptotic expansion. [5M]

5. State and Prove Watson's lemma and hence evaluate $\int_0^5 \frac{e^{-xt}}{1+t^2}dt \; as \; x \to \infty.$ [10M]

6. a) Solve $y' = x + 2y,\; y(0) = 0$ using Euler's method to determine $y(0.4)$ by taking step size $h = 0.1.$ [5M]

b) Apply Runge-kutta method of second order, find the value of y at $x = 0.01,$ given that $\frac{dy}{dx} = x^2 + y$ and $y_0 = 1$ when $x_0 = 0$, by taking $h = 0.01$ as step size. [5M]

7. Find $y(0.1),\; y(0.2), y(0.3)$ from $y' = x^2 - y,\;\; y(0) = 1$ by using Taylor's series method and hence obtain $y(0.4)$ by using Adams-Bashforth method.

[10M]

8. Solve the Poisson's equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square with sides $x = 0 = y,\; x = 3 = y$ with $u = 0$ on the boundary and mesh length equal to 1. Perform 3 iterations using Gauss seidel method. [10M]

9. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(x,0) = \sin \pi x,\; 0 \le x \le 1,$ $u(0,t) = 0 = u(1,t)$. Carryout computations for two levels by taking $h = \frac{1}{3},\; k = \frac{1}{36}.$

[10M]

10. Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = 4\frac{\partial^2 u}{\partial x^2}$, given that $u(0,t) = 0 = u(4,t),\; u_t(x,0) = 0$ and $u(x,0) = x(4-x)$ by taking $h = 1,\; k = 0.5$ up to 4 steps. [10M]

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