Register Number:
Date:

St. Joseph's College (Autonomous), Bangalore
M.Sc Mathematics - I Semester
End Semester Examination: October, 2021
(Examination conducted in January-March 2022)
MT7521: Discrete Mathematics and Graph Theory

Duration: 2.5 Hours **Max. Marks:** 70

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1. The paper contains two pages.
2. Attempt any **SEVEN FULL** questions.

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1. a) A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian, and 14 have taken courses in both French and Russian. If 2092 students have taken at least one of Spanish, French, and Russian, how many students have taken a course in all three languages? **[4m]**

   b) Find the number of positive integers not exceeding 100 that are not divisible by 5 or by 7. **[4m]**

   c) There are 20 married couple in a party. Find the number of ways of choosing 1 woman and 1 man from the party such that the two are not married to each other. **[2m]**

2. a) Explain the Tower of Hanoi puzzle, get the recurrence relation and solve it by iterative approach. **[5 m]**

   b) Define equivalence relation and show that the relation congruent modulo n on set of integers is an equivalence relation. **[5 m]**

3. a) Let $A = \{1, 2, 3, 4, 5\}$ and $M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$ Find the transitive closure of R by Warshall's algorithm . **[5m]**

   b) Define maximal and minimal elements. Find the maximal and minimal elements of the poset $A = (2, 4, 5, 10, 12, 20, 25, /)$ **[5m]**

4. a) Prove that every non-trivial tree contains at least 2 pendant vertices. **[4m]**

   b) A certain tree T of order 13 has only vertices of degree 1,2 and 5. If T has exactly 8 pendent vertices, how many vertices of T have degree 5? **[2m]**

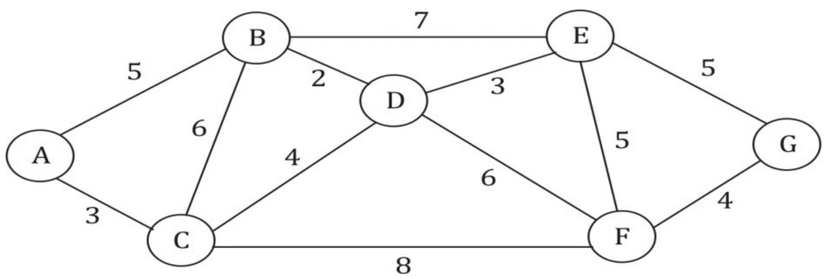

Figure 1: Weighted Graph

c) Explain Prim's algorithm. Find the minimum spanning tree of the graph in Figure 1 using Prim's algorithm. **[4m]**

5. a) Define edge connectivity and vertex connectivity of a graph. **[2m]**

   b) Prove that for every graph G, $\kappa(G) \leq \lambda(G) \leq \delta(G)$ where $\kappa(G)$ is the vertex connectivity, $\lambda(G)$ is the edge connectivity and $\delta(G)$ is the minimum degree of G. **[8m]**

6. a) If G is a graph with 6 vertices, then prove that either G or $\overline{G}$ has a triangle. **[3m]**

   b) Prove that every graph has even number of odd vertices. **[3m]**

   c) Define an Eulerian graph and a Hamiltonian graph with an example each. **[4m]**

7. a) Prove that a digraph D is strong if and only if D contains a closed spanning walk. **[5m]**

   b) If G is a Hamiltonian graph, then prove that for every nonempty proper subset S of vertices of G, $K(G-S) \leq S$ where $K(G)$ is the number of components in graph G. **[3m]**

   c) Define isomorphic graphs. Give an example. **[2m]**

8. State and prove Hall's theorem.

9. State and prove Five color theorem.

10. a) Define the domination number $\gamma(G)$ and the total domination number $\gamma_t(G)$ of a graph G. Prove that $\gamma(G) \leq \gamma_t(G) \leq 2\gamma(G)$. **[5m]**

    b) If G is a graph of order n, then prove that $\frac{n}{1+\Delta(G)} \leq \gamma(G) \leq n - \Delta(G)$ where $\Delta(G)$ is the maximum degree in G. **[5m]**