



Date:

Registration number:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27

M.Sc. Physics - I SEMESTER

SEMESTER EXAMINATION: OCTOBER 2021

(Examination conducted in January-March 2022)

**PH7320- Numerical Techniques**

Time- 2 ½ hrs

Max Marks-70

This question paper contains two printed pages and no parts

**Answer any seven questions. Each question carries 10 marks**

7x10=70

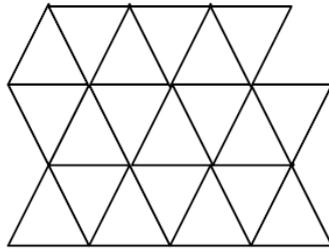
- Using Lagrange interpolation find the slope of the curve at  $x=2$  when the curve passes through the points  $(0,18)$ ,  $(1,10)$ ,  $(3,-18)$  and  $(6,90)$ . (5)
  - The first derivative of a function  $f(x)$  at a particular value of  $x$  can be approximately calculated by  $f'(x) \approx \frac{f(x+h) - f(x)}{h}$  for  $f(x) = 7e^{0.5x}$  and  $h = 0.3$ , find the absolute relative true error at  $x=2$ . (5)
- Use 4 segment Simpson's 1/3 rule to integrate  $f(x) = 0.2+25x+3x^2+2x^4$  between the intervals  $[0,2]$ . Also find the exact value of integral and the relative absolute true error (5)
  - Find the Fourier Cosine transform of  $e^{-x^2}$  (5)
- Find the Fourier transforms for the following. (10)
  - $f(t) = e^{-a|t|}$  (with  $a > 0$ )
  - $f(t) = \delta(t)$
  - $f(t) = 1$
  - $f(t) = \begin{cases} 1 & -T \leq t \leq T \\ 0 & |t| > T \end{cases}$
- The concentration of salt denoted by 'x' in a homemade soap maker as function of time is given by  $\frac{dy}{dx} = 37.5 - 3.5x$ . At the initial time,  $t=0$ , the salt concentration in the tank is 50g/L. Using Euler's method and step size of  $h = 1.5$  min, what is the salt concentration after 3 minutes. Also find the absolute relative true error after solving the above differential equation. (10)
- Given a polynomial with the following data points. (10)

x	1.0	1.1	1.2	1.3	1.4	1.5	1.6
f(x)	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  at  $x = 1.1$  and  $1.5$

- Let  $X$  be a random variable with probability distribution function given (10)
$$f(x) = \begin{cases} cx^2 & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$
find the constant  $c$ , the mean and variance of  $X$  and  $P(X \geq \frac{1}{2})$
- A machine is known to produce 10% defective components. If the random variable  $X$  is the number of defective components produced in a run of 3 components, find the probabilities that  $X$  takes the values 0 to 3. (5)

- b. Consider a random walker on an infinite two-dimensional triangular lattice, a part of which is shown in the figure below. If the probabilities of moving to any of the nearest neighbour sites are equal, what is the probability that the walker returns to the starting position at the end of exactly three steps? (5)



8. a. If electricity power failures occur according to a Poisson distribution with an average of 3 failures every twenty weeks, calculate the probability that there will not be more than one failure during a particular week. (5)
- b. Find the regression line of  $y$  on  $x$  for the following data. (5)

$x$	1	3	4	6	8	9	11	14
$y$	1	2	4	4	5	7	8	9

Estimate the value of  $y$  when  $x = 10$ .

9. In a sample of 1000 cases, the mean of a certain test is 14 and the standard deviation is 2.5. Assuming the distribution to be normal, find (10)
- How many students score between 12 and 15?
  - How many score above 18?
  - How many score below 8?
  - How many score 16?
10. a. A group of scientists working on gender equality reported that the newborns in a certain community within a certain period was 1705 boys and 1527 girls. Do these figures conform to the hypothesis that the sex ratio is  $\frac{1}{2}$ ? (5)
- b. At a certain university, 4% of men are over 6 feet tall and 1% of women are over 6 feet tall. The total student population is divided in the ratio 3:2 in favour of women. If a student is selected at random from among all those six feet tall, what is the probability that the student is a woman? (5)