Date:

Registration number:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
B.Sc. MATHEMATICS - V SEMESTER
SEMESTER EXAMINATION: OCTOBER 2021
(Examination conducted in January-March 2022)

**MT5218 – MATHEMATICS VI**

Time- 2 ½ hrs Max Marks-70

This question paper contains TWO printed pages and THREE parts

## I. Answer any FIVE questions [5*2 =10]

1. Find the real and the imaginary part of $f(z) = \frac{1}{z}, z \neq 0$.
2. Show that $u = e^x \cos y + xy$ is harmonic.
3. Find the fixed points of the transformation $w = \frac{3z-4}{z}$
4. Evaluate $\oint_c \frac{\sin \pi z}{z-\pi}$ , $c: |z+2| = 1$.
5. If $\vec{a}$ is a constant vector, show that $\nabla(\vec{a}.\vec{r}) = \vec{a}$
6. Show that $\vec{F} = (z + \sin y)\hat{\imath} + (x\cos y - z)\hat{\jmath} + (x - y)\hat{k}$ is irrotational.
7. Show that $\vec{v} = (x + 3y)\hat{\imath} + (y - 3z)\hat{\jmath} + (x - 2z)\hat{k}$ is solenoidal.
8. For any scalar field $\phi$, show that $\nabla \times (\nabla\phi) = 0$.

## II. Answer any SEVEN questions [7*6=42]

9. i. Evaluate $\lim_{z \to i} \frac{(z^3+i)}{1-zi}$

   ii. Evaluate $\lim_{z \to e^{\frac{i\pi}{3}}} \frac{z\left(z - e^{\frac{i\pi}{3}}\right)}{z^3+1}$ [2+4]
10. Show that the transformation $w = \frac{1}{z}$ , transforms a circle to a circle.
11. Find the bilinear transformation which maps $0, -i, -1$ on $z$ plane on to $i, 1, 0$ in $w$ plane. Also find its fixed points.
12. Define harmonic function and show that the real and imaginary parts of an analytic function $f(z)$ are harmonic.
13. Find the analytic function when the real part is $\left(r + \frac{1}{r}\right) cos\theta$.
14. Find the orthogonal trajectories of $e^{-x}(x\, siny - y\cos y) = c$.
15. State and prove Cauchy Integral Theorem.

16. Evaluate $\oint_c \frac{z\,dz}{(z^2+1)(z^2-9)}$ , where $c$ is the circle $|z| = 2$.

17. Evaluate $\oint_c \frac{dz}{z\,(z-1)}$ , where $c$ is the circle $|z| = 3$.

**III.** **Answer any THREE questions** **[3*6=18]**

18. i. If $r = |\vec{r}|$ where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then show that $\vec{a} \cdot \left[\nabla\left(\frac{1}{r}\right)\right] = -\frac{\vec{a}.\vec{r}}{r^3}$

ii. Find the scalar field $\phi$, such that $\nabla\phi = (2xy^3z^4)\hat{\imath} + (3x^2y^2z^4)\hat{\jmath} + (4x^2y^3z^3)\hat{k}$.

**[2+4]**

19. Find the constants $a$ and $b$ such that the surfaces $ax^2 - byz = (a+2)x$ and $4x^2y + z^3 = 4$ are orthogonal at the point $(1, -1, 2)$

20. Prove that $div\left(curl\,\hat{F}\right) = 0$.

21. Prove that $\nabla^2\left(\frac{1}{r}\right) = 0$.

22. Show that $\vec{F} = (2xy^2 + yz)\hat{\imath} + (2x^2y + xz + 2yz^2)\hat{\jmath} + (2y^2z + xy)\hat{k}$ is a conservative force field and find its scalar potential.

---