Register Number:

Date:

ST. JOSEPH'S COLLEGE (AUTONOMOUS), BENGALURU-27
B.Sc. MATHEMATICS - I SEMESTER
SEMESTER EXAMINATION: OCTOBER 2021
(Examination conducted in January-March 2022)
MT121: MATHEMATICS I

Duration: 3 Hours **Max Marks:** 100

This question paper contains **TWO** printed pages and **FIVE** parts.

I. ANSWER ANY TEN OF THE FOLLOWING. **(10×2=20)**

1. Find is the Rank of $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$
2. Find the Eigenvalues of $A = \begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix}$
3. State Cayley Hamilton theorem.
4. If a function $f(x)$ is differentiable at $x = a$, then show that it is continuous at $x = a$.
5. Find the n^{th} derivative of $cos(ax + b)$.
6. If $y = x^n logx$ then show that $xy_{n+1} = n!$.
7. State Rolle's theorem.
8. Show that $\frac{x}{1+x} < log(1 + x), \forall x \leq 0$.
9. Find $\lim\limits_{x \to 0} \dfrac{sinx}{x}$.
10. If $u = \tan^{-1}(\frac{y}{x})$ then show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$
11. If $x^3 + y^3 - 3axy = 0$ and find $\frac{dy}{dx}$ using partial differentiation.
12. If $u = \theta(1 + \phi)$ and $v = \phi(1 + \theta)$, show that $\frac{\partial(u,v)}{\partial(\theta,\phi)} = 1 + \theta + \phi$

II. ANSWER ANY FOUR OF THE FOLLOWING. **(4×5=20)**

13. Find the value of a in order that the rank of the matrix A is 3, where $A = \begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ a & 2 & 2 & 2 \\ 9 & 9 & a & 3 \end{bmatrix}$
14. Find the rank of the following matrix by reducing it to normal form. $A = \begin{bmatrix} 0 & 2 & 4 & -4 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$
15. Find the non-trivial solution of the following system
$x + 3y - 2z = 0$
$2x–y + 4z = 0$
$x - 11y + 14z = 0$

16. Examine the consistency and solve if consistent.
$x + 2y - 5z = -13$
$3x - y + 2z = 1$
$2x - 2y + 3z = 2$
$x - y + z = -1$

17. Find the eigen values and eigen vector of $A = \begin{bmatrix} 4 & 1 \\ -1 & 2 \end{bmatrix}$

18. By using Cayley Hamilton theorem find the inverse of $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

III. ANSWER ANY <u>FOUR</u> OF THE FOLLOWING. **(4×5=20)**

19. Show that a function which is continuous in a closed interval attains its bounds atleast once.

20. Discuss the differentiablilty of the function at $x = 0$,
$f(x) = \begin{cases} x^2 sin(\frac{1}{x}) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$

21. Find the n^{th} derivative of **(3+2m)**
   i. $e^{3x} sin^2 x$
   ii. $sinhx.sin2x$

22. Find the n^{th} derivative of $\frac{4x}{(x-1)^2(x+1)}$

23. If $y = (x + \sqrt{1 + x^2})^m$, then show that $(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$.

24. If $y = e^{tan^{-1}x}$ show that $(1 + x^2)y_{n+2} - [2(n + 1)x - 1]y_{n+1} + n(n + 1)y_n = 0$.

IV. ANSWER ANY <u>FOUR</u> OF THE FOLLOWING. **(4×5=20)**

25. Verify Rolle's theorem for $f(x) = x^3 - 4x$ on $[-2, 2]$.

26. State and prove Lagrange's mean value theorem.

27. Verify cauchy mean value theroem for $f(x) = sinx$, $g(x) = cosx$ in $[0, \frac{\pi}{2}]$.

28. Obtain the expansion of $f(x) = \frac{e^x}{1+e^x}$ at $x = 0$ upto x^3.

29. Expand $sinx$ using Taylor's series at $x = \frac{\pi}{2}$

30. Evaluate $\lim_{x \to 0} (1 + x)^{\frac{1}{x}}$.

V. ANSWER ANY <u>FOUR</u> OF THE FOLLOWING. **(4×5=20)**

31. If $u = f(r)$ where $r^2 = x^2 + y^2$ show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(x) + \frac{1}{r} f'(r)$.

32. State and prove Euler's theorem for homogeneous functions.

33. If $u = log(\frac{x^3+y^3}{x-y})$ then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2$

34. If $x = rcos\theta$ and $y = rsin\theta$. Find $J = \frac{\partial(x,y)}{\partial(r,\theta)}$ and $J' = \frac{\partial(r,\theta)}{\partial(x,y)}$.

35. Expand $e^x log(1 + y)$ in Taylor's series(Maclaurin's form) around the origin.

36. Find the critical points of the function and classify them for maxima and minima $f(x, y) = x^3 - 3xy + y^3$.